Examples and Solutions for "Engineering Noise Control" 6th Edition

by

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Equation, Table and Figure numbers refer to the 6$^{\text{th}}$ edition of Engineering Noise Control unless otherwise indicated.

There are many more example problems in the book, "Noise Control: from Concept to Application". Solutions to additional problems at the end of each chapter in that book are also available on www.causalsystems.com.

1

Examples and Solutions, Chapter 1

There are many more relevant examples in Chapter 1 of the book “Noise Control: from Concept to Application”, second edition. There are also relevant problems provided at the end of chapter 1 in that book with corresponding solutions available on www.causalsystems.com.

Example 1.1

Assume that three sounds of different frequencies (or three incoherent noise sources) are to be combined to obtain a total sound pressure level. Let the three sound pressure levels be (a) 90 dB, (b) 88 dB and (c) 85 dB.

Solution 1.1

The solution is obtained by use of Equation (1.86).

For source (a):

$$\langle p_1^2 \rangle = p_{\mathrm{ref}}^2 \times 10^{90/10} = p_{\mathrm{ref}}^2 \times 10 \times 10^8$$

For source (b):

$$\langle p_2^2 \rangle = p_{\mathrm{ref}}^2 \times 6.31 \times 10^8$$

For source (c):

$$\langle p_3^2 \rangle = p_{\mathrm{ref}}^2 \times 3.16 \times 10^8$$

The total mean square sound pressure is:

$$\langle p_t^2 \rangle = \langle p_1^2 \rangle + \langle p_2^2 \rangle + \langle p_3^2 \rangle = p_{\mathrm{ref}}^2 \times 19.47 \times 10^8$$

The total sound pressure level is:

$$L_{pt} = 10 \log_{10}[\langle p_t^2 \rangle / p_{\mathrm{ref}}^2] = 10 \log_{10}[19.47 \times 10^8] = 92.9 \text{ dB re } 20\,\mu\text{Pa}$$

Alternatively, in short form, using Equation (1.101):

$$L_{pt} = 10 \log_{10} \left(10^{90/10} + 10^{88/10} + 10^{85/10}\right) = 92.9 \text{ dB re } 20\,\mu\text{Pa}$$

Some useful properties of the addition of sound pressure levels will be illustrated with two further examples. The following example will show that the addition of two sounds can never result in a sound pressure level more than 3 dB greater than the level of the louder sound.

Example 1.2

Consider the addition of two sounds of sound pressure levels L_1 and L_2 where $L_1 \geq L_2$. Compute the total sound pressure level on the assumption that the sounds are incoherent, and therefore they add on a squared pressure basis.

Solution 1.2

Using Equation (1.101),

$$L_{pt} = 10\log_{10}\left(10^{L_1/10} + 10^{L_2/10}\right)$$

then,

$$L_{pt} = L_1 + 10\log_{10}\left(1 + 10^{(L_2-L_1)/10}\right)$$

Since $L_1 \geq L_2$, it means that the term:

$$10^{(L_2-L_1)/10} \leq 1$$

Replacing the term $10^{(L_2-L_1)/10}$ with its maximum value of 1 gives:

$$L_{pt} \leq L_1 + 3 \text{ dB}$$

Example 1.3

Consider the addition of N sound pressure levels each with an uncertainty of measured level $\pm\Delta$.

$$L_{pt} = 10\log_{10}\left(\sum_{i=1}^{N} 10^{(L_i\pm\Delta)/10}\right)$$

Show that the level of the sum is characterised by the same uncertainty:

$$L_{pt} = 10\log_{10}\left(\sum_{i=1}^{N} 10^{L_i/10}\right) \pm \Delta$$

Evidently, the uncertainty in the total is no greater than the uncertainty in the measurement of any of the contributing sounds.

Solution 1.3

$10^{L+\Delta} = 10^L 10^\Delta$; therefore:

$$L_{pt} = 10\log_{10}\left(10^{(\pm\Delta/10)}\sum_{i=1}^{N} 10^{(L_i)/10}\right) = 10\log_{10}\left(10^{(\pm\Delta/10)}\right) + 10\log_{10}\left(\sum_{i=1}^{N} 10^{(L_i)/10}\right)$$

Thus:

$$L_{pt} = 10\log_{10}\left(\sum_{i=1}^{N} 10^{L_i/10}\right) \pm \Delta$$

Example 1.4

Initially, the sound pressure level at an observation point is due to straight-line propagation and reflection in the ground plane between the source and receiver. The arrangement is altered by introducing a very long barrier, which prevents both initial propagation paths but introduces four new paths (see Section 5.3.5). Compute the noise reduction due to the introduction of the barrier. In situation A, before alteration, the sound pressure level at the observation point is L_{pA} and propagation loss over the path reflected in the ground plane is 5 dB. In situation B, after alteration, the losses over the four new paths are, respectively, 4, 6, 7 and 10 dB.

Solution 1.4

Using Equation (1.105) gives the following result:

$$\begin{aligned}\mathrm{NR} = 10\log_{10}\left[10^{-0/10} + 10^{-5/10}\right] - 10\log_{10}\left[10^{-4/10} + 10^{-6/10} + 10^{-7/10} + 10^{-10/10}\right]\\ = 1.2 + 0.2 = 1.4 \text{ dB}\end{aligned}$$

2

Examples and Solutions, Chapter 2

There are many more relevant examples in Chapter 2 of the book "Noise Control: from Concept to Application", second edition. There are also some relevant problems provided at the end of chapter 2 in that book with corresponding solutions available on www.causalsystems.com.

Example 2.1

Given the 1/3-octave band sound pressure levels measured in a stationary free field and shown in Table 2.1 below, determine the composite loudness in sones and in phons, and determine the 1/3-octave band that contributes most to the loudness.

TABLE 2.1 1/3-octave band levels used for Example 2.1

1/3-octave centre frequency (Hz)	1/3-octave band level (dB)	1/3-octave centre frequency (Hz)	1/3-octave band level (dB)	1/3-octave centre frequency (Hz)	1/3-octave band level (dB)
25	20	31.5	35	40	45
50	83	63	82	80	79
100	77	125	73	160	70
200	69	250	65	315	67
400	63	500	60	630	64
800	61	1000	55	1250	71
1600	72	2000	74	2500	65
3150	61	4000	58	5000	54
6300	52	8000	50	10000	48
12500	46	16000	44		

(a) Use the methods outlined in ANSI/ASA S3.4 (2017), ISO 532-1 (2017) and ISO 532-2 (2017). The computer programs for calculating the loudness on sones and phons using ISO 532-1 (2017) and ISO 532-2 (2017) can be downloaded at no cost from http://standards.iso.org/iso/532/-1/ed-1/en and http://standards.iso.org/iso/532/-2/ed-1/en/, respectively. Software is also available to evaluate the ANSI/ASA S3.4 (2017) method. It can be downloaded from http://hearing.psychol.cam.ac.uk/demos/demos.html. Instructions for using the software are included in the downloaded files.

(b) Use method outlined in ISO 532 (1975) (described above).

Solution 2.1

(a) Run the three programs that evaluate the calculation procedures in the standards, ANSI/ASA S3.4 (2017), ISO 532-1 (2017) and ISO 532-2 (2017). Also do the hand calculations described previously in this section and based on ISO 532 (1975). The results are:

For the Zwicker method (ISO 532-1, 2017):

(i) Loudness level = 94.7 phons

(ii) Loudness = 44.3 sones

(iii) Loudest 1/3 octave band: Not an output available from the software.

For the Moore-Glasberg method (ISO 532-2, 2017):

(i) Monaural loudness level = 90.2 phons

(ii) Binaural loudness level = 95.5 phons

(iii) Monaural loudness = 33.3 sones

(iv) Binaural loudness = 49.6 sones

(v) Loudest 1/3 octave band: Not an output available from the software.

For the Moore-Glasberg method (ANSI/ASA S3.4, 2017):

(i) Monaural loudness level = 86.2 phons

(ii) Binaural loudness level = 95.5 phons

(iii) Monaural loudness = 25.1 sones

(iv) Binaural loudness = 50.2 sones

(v) Loudest 1/3 octave band: Not an output available from the software.

(b) For the ISO 532 (1975) described in this section, we begin by reading from Figure 2.14, the sone level corresponding to each 1/3-octave band level in Table 2.1 on the previous page. The results are listed in Table 2.2 below.

TABLE 2.2 Sone levels corresponding to the 1/3-octave band levels used for Example 2.1

Frequency (Hz)	1/3-oct. level	Sone level	Frequency (Hz)	1/3-oct. level	Sone level	Frequency (Hz)	1/3-oct. level	Sone level
25	20	0	31.5	35	0	40	45	0.16
50	83	7.3	63	82	7.8	80	79	7.2
100	77	7	125	73	6.1	160	70	5.5
200	69	5.5	250	65	4.6	315	67	5.5
400	63	4.6	500	60	4.0	630	64	5.5
800	61	4.9	1000	55	3.6	1250	71	9.9
1600	72	11.1	2000	74	13.5	2500	65	7.8
3150	61	7	4000	58	5.8	5000	54	4.9
6300	52	4.6	8000	50	4.3	10000	48	3.6
12500	46	2.7	16000	44	2.0			

Using Equations (2.33) and (2.34) together with Table 2.2 above, we can calculate the overall sone and phon values as well as the 1/3-octave band with the highest loudness values.

(i) Loudness level = 91.3 phons

(ii) Loudness = 34.9 sones

(iii) Loudest 1/3 octave band: 2000 Hz

Example 2.2

For the purpose of noise control, a rank ordering of loudness may be sufficient. Given such a rank ordering, the effect of concentrated control on the important bands may be determined. A comparison of the cost of control and the effectiveness of loudness reduction may then be possible. In such a case, a shortcut method of rank ordering band levels, which always gives results similar to the more exact method discussed above, is illustrated here. Note that reducing the sound level in dBA does not necessarily mean that the perceived loudness will be reduced, especially for sound levels exceeding 70 dBA. For the spectrum shown in Table 2.3 below, use a shortcut method to rank order the various bands.

TABLE 2.3 Example 2.1 Table

Row description	Octave band centre frequencies (Hz)								
	31.5	63	125	250	500	1000	2000	4000	8000
Band level (dB re 20 µPa)	57	58	60	65	75	80	75	70	65

Solution 2.2

1. Obtain the band loudness in sones for each band from Figure 2.14 in the textbook.
2. Rank the band loudness levels, beginning with 1 for the loudest band.
3. Based on the ranking shown in row 3, enter the adjustment levels shown in row 4 of Table 2.4 below.
4. Add the adjustment levels to the band levels of row 1.
5. Enter adjusted levels in row 5.

Note that the rank ordering is exactly as shown previously in row 3.

TABLE 2.4 Solution 2.2 Table

Row description	Octave band centre frequencies (Hz)								
	31.5	63	125	250	500	1000	2000	4000	8000
1. Band level (dB re 20 µPa)	57	58	60	65	75	80	75	70	65
2. Band loudness index (sones)	0.8	1.3	2.5	4.6	10	17	14	13	11
3. Ranking	9	8	7	6	5	1	2	3	4
4. Adjustment	0	3	6	9	12	15	18	21	24
5. Ranking level	57	61	66	74	87	95	93	91	89

Example 2.3

Given the sound spectrum shown in line 1 of the table below, find the overall unweighted (linear) sound pressure level in decibels and the A-weighted sound pressure level in dBA.

Example 2.3 Table

Octave band centre frequency (Hz)	31.5	63	125	250	500	1000	2000	4000	8000
Linear level (dB)	55	55	50	45	40	41	35	30	25
A-weighting correction (dB)	−39.4	−26.2	−16.1	−8.6	−3.2	0	1.2	1.0	−1.1
A-weighted level (dBA)	15.6	28.6	33.9	36.4	36.8	41.0	36.2	31.0	23.9

Solution 2.3

The linear level is calculated using:

$$L_{pt} = 10 \log_{10} \sum_{i=1}^{9} 10^{L_{pi}/10} = 59.0 \text{ dB}$$

where L_{pi} are the levels shown in line 2 of the table. The A-weighted overall level is found by adding the A-weighting corrections (see Table 2.3 in textbook) to line 2 to obtain line 4 and then adding the levels in line 4 using the above expression to give: $L_{pt} = 44.9$ dBA.

Example 2.4

An Australian timber mill employee cuts timber to length with a cut-off saw. While the saw idles it produces a level of 85 dBA and when it cuts timber it produces a level of 96 dBA at the work position.

If the saw runs continuously and actually only cuts for 10% of the time that it is switched on, compute the A-weighted, 8-hour equivalent sound pressure level.

How much must the exposure be reduced in hours to comply with $L_{\text{EX,8h}} = 85$ dBA?

Solution 2.4

Making use of Equation (2.38) and (2.39) (or Equation (2.77) with $L = 3$, in which case, $L_{\text{EX,8h}} \approx L'_{Aeq,\text{8h}}$), the following can be written:

$$L_{\text{EX,8h}} = 10 \log_{10} \left[\frac{1}{8} \left(7.2 \times 10^{85/10} + 0.8 \times 10^{96/10} \right) \right] = 88.3 \text{ dBA}$$

Let T_a be the allowed exposure time. Then:

$$L_{\text{EX,8h}} = 85.0 \text{ dBA} = 10 \log_{10} \left[\frac{T_a}{8} \left(0.9 \times 10^{85/10} + 0.1 \times 10^{96/10} \right) \right]$$

Solving this equation gives $T_a = 3.7$ hours. The required reduction $= 8 - 3.7 = 4.3$ hours.

Alternatively, use Equation (2.81) and let $L = 3$, $L_B = 85$, which gives:

$$T_a = 8 \times 2^{-(88.34-85.0)/3} = 8/2^{1.11} = 3.7 \text{ hours}$$

Alternatively, for an American worker, $L = 5$ and use of Equation (2.80) gives $L'_{Aeq,8\mathrm{h}} = 87.2$. Equation (2.81) with $L = 5$ and $L_B = 90$ gives for the allowed exposure time T_a:

$$T_a = 8 \times 2^{-(87.2-90.0)/5} = 8 \times 2^{0.56} = 11.8 \text{ hours}$$

Example 2.5

An employee has an exposure level of $L_{\mathrm{exp},A} = 100$ dBA (corresponding to working in an environment where the $L_{Aeq} = 100$ dBA) and wears earplugs with an NRR=29 and earmuffs with an NRR=27. What is the estimated sound pressure level at the ears of the worker when wearing earplugs and earmuffs at the same time according to the OHSA recommendations?

Solution 2.5

The estimated protected level at the ears of the worker is:

$$\begin{aligned} L_{\mathrm{prot},A} &= L_{\mathrm{exp},A} - [0.5 \times (\mathrm{NRR}_h - 7) + 5] \\ &= 100 - [0.5 \times (29 - 7) + 5] = 84 \text{ dBA re } 20\,\mu\mathrm{Pa} \end{aligned}$$

Example 2.6

An employee has an exposure level of $L_{\mathrm{exp},A} = 100$ dBA and wears earmuffs that are labelled with $\mathrm{NRS}_{A,80} = 18$ dB and $\mathrm{NRS}_{A,20} = 32$ dB. What is the estimated sound pressure level at the ears of the worker when wearing earmuffs?

Solution 2.6

The estimated protected level at the ears of the worker is:

$$\begin{aligned} L_{\mathrm{prot},Ax} &= L_{\mathrm{exp},A} - \mathrm{NRS}_{Ax} \\ L_{\mathrm{prot},A,80} &= 100 - 18 = 82 \text{ dBA re } 20\,\mu\mathrm{Pa} \\ L_{\mathrm{prot},A,20} &= 100 - 32 = 68 \text{ dBA re } 20\,\mu\mathrm{Pa} \end{aligned}$$

Hence, $L_{\mathrm{prot},A,80} = 82$ dBA is the protected A-weighted level most users will not exceed, and $L_{\mathrm{prot},A,20} = 68$ dBA is the protected A-weighted level only a few motivated well-trained users will not exceed.

Example 2.7

An employee has A-weighted and C-weighted exposure levels of $L_{\mathrm{exp},A} = 102$ dBA and $L_{\mathrm{exp},C} = 107$ dB(C), respectively. The employee wears earplugs rated as having an SNR=34 dB, H=33 dB, M=31 dB and L=30 dB. What is the protected sound pressure level when the worker is wearing earplugs?

Solution 2.7

Step 1: Calculate the difference between the C- and A-weighted exposure levels using Equation (2.87) as:

$$\Delta = 107 - 102 = 5 \text{ dB}$$

Step 2: The value of $\Delta > 2$ dB, so calculate the PNR value using Equation (2.89) as:

$$\begin{aligned}
\text{PNR} &= \text{M} - \left[\frac{\text{M} - \text{L}}{8} \times (L_{\exp,C} - L_{\exp,A} - 2)\right] \\
&= 31 - \left[\frac{31 - 30}{8} \times (107 - 102 - 2)\right] \\
&= 30.6 \text{ dB}
\end{aligned}$$

Step 3: Calculate the A-weighted protected level using Equation (2.90) as

$$L_{\text{prot},A} = L_{\exp,A} - \text{PNR}$$

$$= 102 - 30.6 = 71.4 \ \text{ dBA re } 20\,\mu\text{Pa}$$

Example 2.8

A worker has a C-weighted exposure level of $L_{\exp,Ceq,8\text{h}} = 100$ dB(C). It is desired to have a protected noise level at the ears of the worker when wearing hearing protection of $L_{\text{prot},Aeq,8\text{h}} =$ 80 dBA. What SLC_{80} value is required for the hearing protection device, and what Class rating is suitable?

Solution 2.8

Using Equation (2.91), the required SLC_{80} value is:

$$\text{SLC}_{80} = L_{\exp,Ceq,8\text{h}} - L_{\text{prot},Aeq,8\text{h}}$$

$$= 100 - 80 = 20 \text{ dB}$$

Hence, the required rating of the hearing protector is $\text{SLC}_{80} = 20$, which according to Table 2.11 in the textbook has a Class 3 rating.

Example 2.9

Find the NR number for the sound spectrum of Example 2.1.

Solution 2.9

Plot the unweighted sound spectrum on a set of NR curves. The highest curve that envelopes the data is NR = 81 (interpolated between the NR80 and NR85 curves).

Example 2.10

A vintage musical instrument collector finds playing his steam calliope a relaxing exercise. He lives in a generally urban area with infrequent traffic. When he plays, the resulting sound pressure level due to his instrument ranges to about 55 dBA at the nearby residences. If he finds himself insomnious at 3 am, should he play his calliope as a sedative to enable a return to sleep?

Solution 2.10

Begin with the base level of 40 dBA and add 5 dBA to account for the tonal nature of the sound. Next, add the adjustments indicated by Table 2.26 in the the textbook for the time of day and location. The following corrected criterion is obtained:

$$35 - 10 + 5 = 30 \text{ dBA}$$

The amount by which the expected level exceeds the corrected criterion at the nearby residences is:

$$55 - 30 = 25 \text{ dBA}$$

Comparison of this level with the levels shown in Table 2.27 in the textbook suggests that strong public reaction to his playing may be expected. He had best forget playing as a cure for his insomnia!

3

Examples and Solutions, Chapter 3

There are a number of relevant examples in Chapter 2 of the book “Noise Control: from Concept to Application”, second edition which also apply to this chapter. There are also some relevant problems provided at the end of chapter 2 in that book with corresponding solutions available on www.causalsystems.com.

4

Examples and Solutions, Chapter 4

There are many more relevant examples in Chapter 3 of the book "Noise Control: from Concept to Application", second edition. There are also relevant problems provided at the end of chapter 3 in that book with corresponding solutions available on www.causalsystems.com.

Example 4.1

A swimming pool pump has a sound power level of 60 dB re 10^{-12} W when resting on the ground in the open. It is to be placed next to the wall of a building, a minimum of 2 m from a neighbour's property line. If the pump ordinarily radiates equally well in all directions (omnidirectional), what sound pressure level do you expect at the nearest point on the neighbour's property line?

Solution 4.1

It is implicitly assumed that the source is well within one-quarter of a wavelength of the reflecting wall and ground to calculate an upper bound for the expected sound pressure level.

As an upper bound, a constant volume-velocity source is assumed. Previous discussion has shown that, for a source of this type, the radiated sound power level, L_{W0}, in the presence of no reflecting planes is 3 dB less than it is in the presence of one reflecting plane. Thus, the free-field sound power in the absence of any reflecting plane is:

$$L_{W0} = 60 - 3 = 57 \text{ dB}$$

Use is made of Equation (4.133) to write:

$$L_p = 57 + 20\log_{10} 4 - 10\log_{10}(16\pi)$$

$$L_p = 52 \text{ dB re } 20\,\mu\text{Pa}$$

Alternatively, it may be assumed that the sound power is not affected by the reflecting surfaces (either by assuming a constant power source or by assuming that the source is more than one-half of a wavelength from the reflecting surfaces) and use is made of Equation (4.131) to calculate a lower bound as:

$$L_p = 60 + 10\log_{10} 4 - 10\log_{10}(16\pi)$$

$$L_p = 49 \text{ dB re } 20\,\mu\text{Pa}$$

It is concluded that a sound pressure level between 49 and 52 dB may be expected at the neighbour's property line. Note that if a constant acoustic-pressure aerodynamic source were assumed and its acoustic centre was within one-tenth of a wavelength from the reflecting surfaces, the lower bound would be 46 dB. However, in this case, the assumption of a constant acoustic-pressure source is not justified.

Example 4.2

Sound pressure levels are measured at 20 points on a hemispherical surface surrounding a machine placed on a reflecting plane, at locations specified in ISO 3744. The following table gives the levels measured in the 500 Hz octave band, in dB re 20 µPa. Find L_W, the sound power level radiated by the machine, if the radius of the test hemisphere is 3 m. Assume a room temperature of 20°C, a relative humidity of 50% and an atmospheric pressure of 101325 Pa.

Example 4.2 Table

Measurement position number	L_{pi} (dB)	$10^{L_{pi}/10}$	Measurement position number	L_{pi} (dB)	$10^{L_{pi}/10}$
1	60	1.000E+06	11	55	3.162E+05
2	60	1.000E+06	12	55	3.162E+05
3	62	1.585E+06	13	59	7.943E+05
5	55	3.162E+05	14	58	6.310E+05
5	58	6.310E+05	15	57	5.012E+05
6	57	5.012E+05	16	58	6.310E+05
7	58	6.310E+05	17	55	3.162E+05
8	59	7.943E+05	18	62	1.585E+06
9	55	3.162E+05	19	60	1.000E+06
10	55	3.162E+05	20	60	1.000E+06
Total					1.418E+07

Solution 4.2

All points in the array are associated with equal areas.
Using Equation (4.138), the following is obtained:

$$L_p = 10\log_{10}\left[\frac{1}{20}\sum_{1}^{20} 10^{(L_{pi}/10)}\right] \qquad \text{(dB re 20 µPa)}$$

Using the above Example table:

$$L_p = 10\log_{10}(1.418\times 10^7/20) = 10\log_{10}(7.091\times 10^5) = 58.51 \text{ dB re 20 µPa}$$

The sound power level may be calculated using Equation (4.139), where:
$C_1 = 10\log_{10}[400/(\rho c)] = 10\log_{10}[400/(1.206\times 342.9)] = -0.14$
$C_2 = -10\log_{10}\left(\dfrac{P_s}{101325}\right) + 15\log_{10}\left(\dfrac{273+T}{296}\right) = -10\log_{10}(1) + 15\log_{10}(295/296) \approx -0.02$
From Table 5.3 in the textbook, the atmospheric absorption coefficient at 500 Hz, $\alpha_a = 2.7$ dB per 1000 m.
$C_3 = A_a$, which according to Equation (5.77) gives:

$$C_3 = 0.0027\times 3\left[1.0 + \frac{\left(10^{(3/20)} - 10^{(-3/20)}\right)^2}{10}(1.0 - 0.2303\times 0.0027\times 3)\right]^{1.6} = 0.015$$

≈ 0.0 dB.

Substituting values for L_p, r, C_1, C_2 and C_3 into Equation (4.139) gives:
$L_W = 58.5 + 20\log_{10} 3 + 8 - 0.1 = 76$ dB re 10^{-12} W, which is the sound power level of the machine.

Example 4.3

A machine is located in a semi-reverberant workshop area at the junction of a concrete floor and brick wall. The average sound pressure level in the 1000 Hz octave band over the test surface (a quarter sphere) is 82 dB at a radius of 2 m and 80 dB at a radius of 5 m. Determine the sound power level in the 1000 Hz octave band for the machine, assuming that the sound pressure level measurements were made in the far field of the source.

Solution 4.3

$$L = L_{p1} - L_{p2} = 2 \text{ dB}$$

$$S_1^{-1} = 1/(\pi r_1^2) = 0.0796 S_2^{-1} = 1/(\pi r_2^2) = 0.0127$$

$$S_1^{-1} - S_2^{-1} = 0.0668$$

Therefore, using Equation (4.155):

$$L_W = 80 - 10\log_{10}(0.0668) + 10\log_{10}(10^{0.2} - 1) - 0.15$$

$$= 80 + 11.75 - 2.33 - 0.15 = 89 \text{ dB re } 10^{-12}\text{W}$$

Example 4.4

A subsonic jet has the directivity pattern shown in the following table. Calculate the sound pressure levels at 1 m from a small jet of sound power 100 dB re 10^{-12} W.

Solution 4.4

Use Equations (4.131) and (4.132) and write $L_p = 100 - 11 + D$.
Thus, $L_p = 89 + D$.
The last column in the table is constructed by adding 89 dB to the numbers shown in the second column.

Table of exhaust jet directivities for Example 4.4

Angle relative to direction of jet axial velocity (degrees)	Directivity index DI (dB)	Predicted sound pressure level at 1 m (dB re 20 µPa)
0	0.0	89
15	3.0	92
30	5.0	94
45	2.5	91.5
60	−1.0	88
75	−4.0	85
90	−6.0	83
105	−7.5	81.5
120	−8.0	81
150	−9.0	80
180	−10.0	79

5

Examples and Solutions, Chapter 5

There are many more relevant examples in Chapters 4 and 7 of the book "Noise Control: from Concept to Application", second edition. There are also some relevant problems provided at the end of chapters 4 and 7 in that book with corresponding solutions available on www.causalsystems.com.

Example 5.1

Calculate the excess attenuation due to ground effects for the 1000 Hz octave band, given the values for discrete frequencies in the band shown in the following table.

Example 5.1 Table

Frequency (Hz)	A_{gi} (dB)
710	5
781	7
852	9
923	11
994	17
1065	11
1136	9
1207	7
1278	5
1349	4
1420	3

Solution 5.1

The band-averaged attenuation is obtained by combining the tabulated values logarithmically, as:

$$A_g = -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n} 10^{-A_{gi}/10}\right] = 6.7 \text{ dB}$$

Example 5.2

A point source of low-frequency, broadband sound at 1 m above the ground introduces unwanted noise at a receiver, also 1 m above the ground at 4 m distance. The ground surface is rough grassland. What is the effect, in the 250 Hz octave band, on the receiver, of a barrier centrally located, 2 m high and 6 m wide?

Solution 5.2

As octave bands are considered, waves reflected from the ground are combined incoherently with non-reflected waves. In fact, all sound waves arriving at the receiver by different paths are combined incoherently.

1. Calculate the reflection coefficient of the ground.
The flow resistivity, R_1, for rough grassland is between 10^5 and 3×10^5 MKS rayls/m so we will choose the mid-point of the range; that is, 2×10^5 MKS rayls/m (see Table 5.1 in the textbook). This results in a value of flow resistance parameter, $\rho f/R_1 = 1.5 \times 10^{-3}$. For the geometry described, the angle of incidence from the horizontal for the ground reflection of the ray diffracted over the top of the barrier is 56°(for both reflection on the source side and reflection on the receiver side), and 15.5° for the waves diffracted around the ends. The reflection losses, $A_{rf,2}$ and $A_{rf,7}$, corresponding to these angles of incidence (56° and 15.5°, respectively) and flow resistance parameter 1.5×10^{-3}, are 1.3 dB and 5.0 dB, respectively (see Figure 5.2 in the textbook).

2. Next, calculate the sound pressure level at the receiver due to each diffracted path.
For each diffracted path calculate the required path lengths ($A_i + B_i$ and d_{SRi}, see Figure 5.18 in the textbook), Fresnel number, N_i (Equation (5.119)) and attenuation, $A_{b,i}$ (Figure 5.17 and Equation (5.121) in the textbook). Add the reflection loss, $A_{rf,i}$ (where appropriate), to obtain the total attenuation, $A_{b,i} + A_{rf,i}$.

(a) Reflected waves over the top (three paths):

Image source–receiver path:
$A_{b,2} = 15.8 + 20\log_{10}[5.8/4.5] = 18.0$ dB; $A_{rf,2,2} = 1.3$ dB; $A_{b,2} + A_{rf,2} = 19.3$ dB
$d_{\mathrm{SR}2} = 2\sqrt{5} = 4.5$ m; $A_2 + B_2 = \sqrt{13} + \sqrt{5} = 5.8$ m; $N_2 = 1.9$;

Source–image receiver path (same as above): $A_{b,3} + A_{rf,3} = 19.3$ dB

Image source–image receiver path:
$d_{\mathrm{SR}4} = d_{\mathrm{SR}1} = 4$ m; $A_4 + B_4 = 2\sqrt{13} = 7.2$ m; $N_4 = 4.7$;
$A_{b4} = 19.8 + 20\log_{10}[7.2/4] = 24.9$ dB; $A_{rf,4} = 2 \times 1.3 = 2.6$ dB; $A_{b4} + A_{rf,4} = 27.5$ dB

(b) Reflected waves around barrier ends (two paths):

Image source–receiver path (using Equations (5.120) and (5.121)):
$d_{\mathrm{SR}7} = d_{\mathrm{SR}8} = 4.5$ m; $A_7 + B_7 = A_8 + B_8 = 2\sqrt{14} = 7.5$ m; $N_7 = N_8 = 4.3$;
$A_{b,7} = A_{b,8} = 19.5 + 20\log_{10}[7.5/4.5] = 23.9$ dB; $A_{rf,7} = A_{rf,8} = 5$ dB;
$A_{b,7} + A_{rf,7} = A_{b,8} + A_{rf,8} = 28.9$ dB

(c) Non-reflected waves (three paths):

Source–receiver path over top of barrier:
$d_{\mathrm{SR}1} = 4$m; $A_1 + B_1 = 2\sqrt{5} = 4.5$ m; $N_1 = 0.7$;
$A_{b,1} = 12.0 + 20\log_{10}[4.5/4] = 13.0$ dB

Source–receiver path around barrier ends (two paths):
$A_5 = A_6 = B_5 = B_6 = \sqrt{13}$
$d_{\mathrm{SR}5} = d_{\mathrm{SR}6} = 4$ m; $A_5 + B_5 = A_6 + B_6 = 2\sqrt{13} = 7.2$ m; $N_5 = N_6 = 4.6$;
From Equation (5.121), $A_{b_5} = A_{b_6} = 19.8 + 20\log_{10}[7.2/4] = 24.9$ dB

Using Equation (1.105), combine all attenuations for each of the eight paths with the barrier present (19.3 dB, 19.3 dB, 27.5 dB, 28.9 dB, 28.9 dB, 13 dB, 24.9 dB and 24.9 dB) ($NR_{B,i}$ in

Equation (1.105)) and the attenuations of the two paths with the barrier absent (direct path, $NR_{A,1} = 0$ and reflected path, $NR_{A,2} = A_{rf,w} = 3$ dB in Equation (1.105) – see Figure 5.2 with $\beta = 27°$), to give an overall attenuation of approximately 12.5 dB (usually given as 12 dB). This is the same value that would have been obtained if all ground reflections were ignored, and only the diffraction over the top and around the ends of the barrier were considered. If only diffraction over the top were considered, the result of 13 dB would only be half a dB greater. In general, results of acceptable accuracy are often obtained by considering only diffraction over the top of the barrier and ignoring ground reflection, provided that the barrier has reasonable dimensions. Note that the final result of the calculations is often given to the nearest dB because this is the best accuracy that can be expected in practice and the accuracy with which Figure 5.17 may be read is in accord with this observation.

6

Examples and Solutions, Chapter 6

Example problems and solutions relevant to this chapter are provided in Chapter 6 of the book "Noise Control: from Concept to Application", second edition. There are also relevant problems provided at the end of chapter 6 in that book with corresponding solutions available on www.causalsystems.com.

7

Examples and Solutions, Chapter 7

There are many more relevant examples and solutions provided in Chapter 7 of the book "Noise Control: from Concept to Application", second edition. There are also relevant problems provided at the end of chapter 7 in that book with corresponding solutions available on www.causalsystems.com.

Example 7.1

A double gypsum board wall is mounted at the perimeter in an opening of dimensions 3.0 × 2.44 m in a test facility. The spacing between the panels is 0.1 m. The surface densities and critical frequencies of each panel are, respectively, 12.16 kg/m^2 and 2500 Hz. Calculate the expected transmission loss using Sharp's theory. The space between the walls is well-damped with a 50 mm thick layer of sound-absorbing material. However, the panels themselves have not been treated with damping material.

Solution 7.1

Reference is made to Figure 7.11. Calculate the coordinates of point A:

$$f_0 = 80.4\sqrt{2 \times 12.16/(0.1 \times 12.16^2)} = 103 \text{ Hz}$$

$$\text{TL}_A = 20\log_{10}(2 \times 12.16) + 20\log_{10} 103 - 48 = 20 \text{ dB}$$

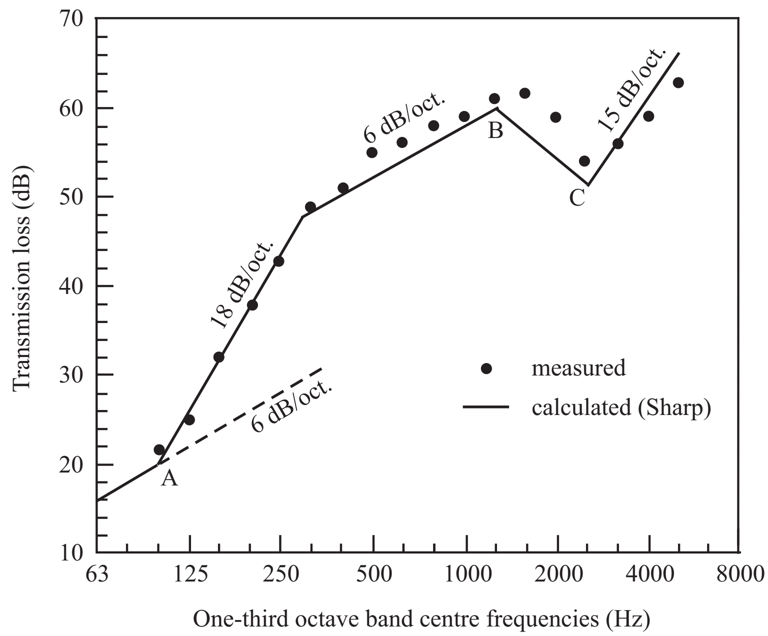
Calculate the coordinates of point B. Since the panel is only supported at the edge, the area associated with each support is less than half of that assumed in the theory; and for this reason we empirically add 4 dB to the calculated transmission loss at point B. As there is sound-absorbing material in the cavity, $\text{TL}_B = \text{TL}_{B2}$

$$\begin{aligned}\text{TL}_{B2} =&20\log_{10} 12.16 + 10\log_{10} 2.44 + 30\log_{10} 2500\\ &+ 20\log_{10} 2 - 78 + 4 = 60 \text{ dB};\end{aligned}$$

Thus TL_B =60 dB at a frequency of 2500/2 = 1250 Hz.
Calculate the coordinates of point C. In the absence of better information, assume a loss factor $\eta = 0.1$ for each panel:

$$\text{TL}_c = 60 + 6 - 10 - 5 = 51 \text{ dB}$$

Construct the estimated transmission loss curve shown in the following figure.



Example 7.1 figure.

Example 7.2

Calculate the overall transmission loss at 125 Hz of a wall of total area 10 m^2 constructed from a material that has a transmission loss of 30 dB; if the wall contains a door of area 3 m^2, constructed of a material having a transmission loss of 10 dB.

Solution 7.2

For the main wall, the transmission coefficient is:

$$\tau_1 = 1/[10^{(30/10)}] = 0.001$$

while for the panel:

$$\tau_2 = 1/[10^{(10/10)}] = 0.100$$

Hence the overall transmission coefficient is (see Equation (7.114):

$$\tau = \frac{(0.001 \times 7) + (0.100 \times 3)}{10} = 0.0307$$

The overall transmission loss is therefore:

$$\mathrm{TL_{av}} = 10 \log_{10}(1/0.0307) = 15 \text{ dB}$$

Example 7.3

A small pump has a sound power level of 80 dB re 10^{-12} W. It is to be contained in an enclosure of 2.2 m^2 surface area. Its sound power spectrum peaks in the 250 Hz octave band, and rolls off above and below at 3 dB per octave. Calculate the predicted sound pressure level in octave bands from 63 Hz to 8 kHz at the outside surface of the enclosure, assuming average acoustical conditions within the enclosure and a wall transmission loss (TL), as shown in the following example table for each of the octave bands.

Calculations for Example 7.3

	Octave band centre frequency (Hz)							
	63	125	250	500	1000	2000	4000	8000
Wall TL (dB)	8	11	12	15	18	23	25	30
Correction, C (dB)	13	11	9	7	5	4	3	3
Relative power spectrum (dB)	−6	−3	0	−3	−6	−9	−12	−15
L_W (dB re 10^{-12} W)	69.6	72.6	75.6	72.6	69.6	66.6	63.6	60.6
L_p (dB re 20 µPa)	71	69	69	61	53	44	38	30

Solution 7.3

From Table 7.13 in the textbook, for average conditions, enter values for C in the table above. The relative power spectrum, as given, is shown in row three of the above table. Logarithmic addition (see Section 1.11.3 in the textbook) of the values shown in row three gives the sum as 4.4 dB. The total is required to equal 80 dB; thus absolute levels in each band are determined by adding to the relative levels: $80 - 4.4 = 75.6$ dB. The resulting band sound power levels, L_W, are given in the table in row four.

Use Equation (7.124) in the textbook to calculate the required sound pressure levels.

$$S_E = 2.2 \text{ m}^2; \qquad 10\log_{10} S_E = 3.4 \text{ dB}$$

Therefore, assuming that $\rho c = 400$:

$$L_{p1} = L_W - 3.4 - \text{TL} + C$$

The estimates of L_{p1} based on the above equation are indicated in the last row of the above example table.

Example 7.4

A small personnel enclosure of nominal dimensions 2 m wide, 3 m long and 2.5 m high is to be constructed of single leaf brick 125 mm thick, plastered on both sides. The floor will be of concrete but the ceiling will be of similar construction to the walls (not bricks but plastered, similar weight, etc.). Determine the expected noise reduction (NR) for the basic hard wall design. Assume that the external sound field is essentially reverberant and that any direct field from the source that is incident on the enclosure is negligible.

Solution 7.4

Use Tables 6.2 and 7.12 in the textbook to determine values of wall and ceiling absorption coefficients and transmission loss. Enter the values in Example 7.4 table on the next page.
Calculate:

$$\begin{aligned} S_i\bar{\alpha}_i &= [2(2 \times 2.5 + 3 \times 2.5) + 2 \times 3]\bar{\alpha}_w + [2 \times 3]\bar{\alpha}_f \\ &= 31\bar{\alpha}_w + 6\bar{\alpha}_f \end{aligned}$$

Enter the values in the Example 7.4 table. Calculate $10\log_{10}(S_E/S_i\bar{\alpha}_i)$ and enter it in the table.

Calculate the external surface area excluding the floor: $S_E = 31$ m^2.
Calculate noise reduction, NR, using Equation (7.16) in the textbook rather than Equations (7.132) and (7.133), as $\bar{\alpha}$ is small. In this case, the former test partition area, A, becomes the external area, S_E, exposed to the external sound field:

$$\mathrm{NR} = \mathrm{TL} - 10\log_{10}\left[\frac{S_E}{S_i\bar{\alpha}_i}\right]$$

The results are entered in Example 7.4 table.

Calculations for Example 7.4

	Octave band centre frequency (Hz)							
	63	125	250	500	1000	2000	4000	8000
TL (Table 7.12 in textbook)	30	36	37	40	46	54	57	59
$\bar{\alpha}_w$ (Table 6.2 in textbook)	0.013	0.013	0.015	0.02	0.03	0.04	0.05	0.06
$\bar{\alpha}_f$ (Table 6.2 in textbook)	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.03
$S_i\bar{\alpha}_i$ (m)	0.463	0.463	0.525	0.68	1.05	1.36	1.67	2.04
$S_E/S_i\bar{\alpha}_i$	67.0	67.0	59.0	45.6	29.5	22.8	18.6	15.2
$10\log_{10}(S_E/S_i\bar{\alpha}_i)$	18	18	18	17	15	14	13	12
NR (dB)	12	18	19	23	31	40	44	47

Example 7.5

Suppose that an opening of 0.5 m^2 is required for ventilation in the enclosure wall of Example 7.4. What transmission loss (TL) must any muffling provide for the ventilation opening if the noise reduction (NR) of the enclosure must not be less than 16 dB in the 250 Hz octave band?

Solution 7.5

Assume that $S_i\bar{\alpha}_i$ is essentially unchanged by the small penetration through the wall; then from the previous example at 250 Hz, $10\log_{10} S_i\bar{\alpha}_i = -2.8$ and $10\log_{10} S_E = 14.9$ dB.
In Equation Equation (7.16) in the textbook, replace A with S_E; then NR = TL $+10\log_{10} S_i\bar{\alpha}_i - 10\log_{10} S_E$. Let NR = 16. Then TL = 16 + 2.8 + 14.9 = 33.7 dB. Thus, the net TL required is 33.7 dB.
Calculate the transmission coefficient τ of the ventilation opening.
Equation (7.114) in the textbook gives:

$$31\tau = (31 - 0.5)\tau_w + 0.5\tau_v$$

Using Example 7.4 table above and Equation (7.13) in the textbook:

$$\tau_w = 10^{-37/10} = 0.000200$$

Putting the above in Equation (7.13) in the textbook gives:

$$\mathrm{TL} = -10\log_{10}\tau = -10\log_{10}\left(\frac{30.5}{31}\times 0.0002 + \frac{0.5}{31}\tau_v\right) = 33.7\ \text{(dB)}$$

From the above, the following is obtained:

$$\frac{30.5}{31}\times 0.0002 + \frac{0.5}{31}\tau_v = 10^{-3.37} = 0.00043$$

or

$$\tau_v = \frac{31}{0.5}\left(0.00043 - \frac{30.5}{31} \times 0.0002\right) = \frac{31}{0.5}(0.000233) = 0.0145$$

Calculate the required TL of muffling for the vent:

$$\begin{aligned} \mathrm{TL}_v &= -10\log_{10}\tau_v \\ &= -10\log_{10} 0.0145 = 18.0 \text{ (dB)} \end{aligned}$$

Example 7.6

Find the transmission loss at 1200 Hz of a solid door, 2.10 × 0.9m, with a 12.5 mm gap along its bottom edge. The door itself has a transmission loss of 30 dB (τ=0.001).

Solution 7.6

From Figure 7.14 in the textbook, a 25 mm gap has a transmission coefficient, $\tau = 0.3$ at 1200 Hz. (Note that, because the floor on either side of the door acts as a plane reflector, the effective width of the gap is doubled when Figure 7.14 is used but not when the area of the gap is calculated.):

$$\begin{aligned} \bar{\tau} &= \frac{(0.001 \times 2.10 \times 0.90) + (0.3 \times 0.90 \times 0.0125)}{(2.10 \times 0.90) + (0.90 \times 0.0125)} \\ &= \frac{0.00189 + 0.003375}{1.890 + 0.01125} \\ &= 0.00277 \end{aligned}$$

$$\mathrm{TL} = 10\log_{10}(1/0.00277) = 25.6 \text{ (dB)}$$

Example 7.7

Given an omnidirectional noise source with a sound power level of 127 dB re 10^{-12} W in the 250 Hz octave band, calculate the sound pressure level in the 250 Hz octave band inside a building situated 50 m away. One side of the building faces in the direction of the sound source and all building walls, including the roof (which is flat), have an average field incidence transmission loss of 20 dB in the 250 Hz octave band. The building is rectangular in shape. Assume that the total Sabine absorption in the room (5 m × 5 m × 5 m) in the 250 Hz octave band is 15 m^2. Also assume that the ground between the source and building is acoustically hard (for example, concrete), and the excess attenuation due to ground effect is −3 dB.

Solution 7.7

First of all, calculate the sound pressure level incident, on average, on each of the walls and roof. Without the building, the sound pressure level 50 m from the source is determined by substitution of Equation (5.64) in the textbook, for a point source, into Equation (5.62), as follows:

$$L_p = 127 - 20\log_{10} 50 - 11 + 3 = 85 \text{ dB re } 20\,\mu\text{Pa}$$

Next, calculate the incident sound power on each wall using Equations (1.78), (1.85), (1.86) and (1.88) with the assumption, $\rho c \approx 413$.

Wall facing the source:

$$L_W = 85 + 10\log_{10} 25 - 10\log_{10} \rho c = 98.9 \text{ dB re } 10^{-12}\text{W}$$

Walls adjacent to the one facing the source, and the ceiling:
The effective Fresnel number (see Equation (5.119) in the textbook) due to the barrier effect of the building is zero, resulting in a 5 dB reduction in sound pressure level due to diffraction. Thus:

$$L_W = 98.9 - 5 = 93.9 \text{ dB re } 10^{-12}\text{W}$$

Wall opposite the source:
The effective Fresnel number (see Equation (5.119) in the textbook) due to the barrier effect of the building varies from $N = (2/\lambda)(2.5)$ for sound diffracted to the centre of the wall ($A \approx d,\ B = 0.5$) to zero ($A \approx d,\ B = 0.0$) for sound diffracted to the edges of the wall. From Figure 5.17 in the textbook, this corresponds to a noise reduction ranging from 18 dB to 5 dB. The additional effect due to the finite thickness of the building is calculated using Equation (5.156) in the textbook and is approximately 1 dB. Thus the area-weighted average noise reduction for the sound incident on this wall is approximately 12 dB.
Thus $L_W = 98.9 - 12 = 86.9$ dB re 10^{-12} W.
As there are 3 diffracted paths around the building for sound arriving at the rear wall, the power level on the rear wall should be increased by 10log10(3)=4.8 dB, so the total power on the rear wall becomes 91.7 dB.

Next, calculate the total power radiated into the enclosure.

Wall facing the source:
As the sound field is normally incident, the normal incidence transmission loss must be used. For a 1/3-octave band of noise, $\text{TL}_N = \text{TL} + 5.6$ (Equation (7.59) in the textbook). Thus the power radiated into the enclosure is from this wall is:
$L_W = 98.9 - 20 - 5.6 = 73.3$ dB.

All other walls and ceiling (4 total):
For these the transmission loss to be used is the field incidence transmission loss. Thus the total sound power radiated into the room by these surfaces is the logarithmic sum (see Section **??**) of 73.9 dB, 73.9 dB, 73.9 dB and 71.7 dB, and is equal to 79.5 dB. The total power radiated into the room is the logarithmic sum of the front wall contribution (73.3 dB) and all other walls (79.5 dB), and is equal to 80.4 dB.

The total sound absorption in the room is 15 m^2. The total room surface area $= 6 \times 5 \times 5 = 150$ m^2. Assuming that there is an additional 5% area due to equipment in the room, the average sound absorption coefficient in the room is: $\bar{\alpha} = 15/(150 \times 1.05) = 0.095$.

The sound pressure level in the room is given by Equation (7.136) in the textbook as:

$$L_p = 80.4 + 10\log_{10}\left(\frac{1}{125} + \frac{4 \times (1 - 0.095)}{15}\right) \approx 74 \text{ dB re } 20\,\mu\text{Pa}$$

An accuracy greater than ± 1 dB is not warranted due to the approximations made in the calculations.

8

Examples and Solutions, Chapter 8

There are many more relevant examples in Chapter 8 of the book "Noise Control: from Concept to Application", second edition. There are also relevant problems provided at the end of chapter 8 in that book with corresponding solutions available on www.causalsystems.com.

Example 8.1

Consider a QWT muffler attached to a main duct, as shown in Figure 8.16(a) in the textbook, where the dimensions and parameters of the system are listed in Table 8.3 in the textbook. Calculate the transmission loss of the muffler, neglecting damping and using .

TABLE 8.1 Parameters used in the analysis of a circular duct with a QWT

Description	**Parameter**	**Value**	**Units**
Diameter main duct	D_1	0.1	m
Diameter QWT	D_2	0.05	m
Length QWT	L_{QWT}	1.5	m
Speed of sound	c	343.24	m/s
Density	ρ	1.2041	kg/m^3
Velocity at inlet	u_1	0.001	m/s

Solution 8.1

The geometric length of the QWT is $L_{\text{QWT}} = 1.5$ m. As described in Section 8.8.2.1 in the textbook, when determining the effective length of the QWT, ℓ_e, it is necessary to add an additional length called the end correction, ℓ_0. For the QWT in this example, where the ratio of the QWT diameter and the main duct diameter is $\xi = 0.05/0.1 = 0.5$, the end correction can be estimated using Equation (**??**) as:

$$\ell_0 = a(0.9326 - 0.6196\xi) = (0.05/2) \times (0.9326 - 0.6196 \times 0.5) = 0.01557 \text{ m}$$

Hence the effective acoustic length of the quarter-wavelength resonator is:

$$\ell_e = L_{\text{QWT}} + \ell_0 = 1.5 + 0.01557 = 1.516 \text{ m}$$

The lowest frequency at which the maximum TL is expected to occur is at:

$$f_{\text{QWT}} = \frac{c}{4 \times \ell_e} = \frac{343.24}{4 \times 1.516} = 56.6 \text{ Hz} \tag{8.1}$$

The impedance of the resonator, $Z_{\rm side}$, comprises the sum of the impedance of the throat, Z_t, and the cavity of the resonator, Z_c. For no mean flow through the main duct, one can assume $Z_t = 0$. The impedance of the quarter-wavelength resonator is:

$$Z_{\rm side} = 0 + Z_c = -{\rm j}\frac{\rho c}{S}\cot(k\ell_e) \tag{8.2}$$

Using Equation (8.141) and (8.142) in the textbook, the TL versus frequency is given by:

$$\begin{aligned} {\rm TL} &= 20\log_{10}\left[\frac{1}{2}\left|1 + 0 + \frac{\rho c}{S_{\rm u}}\frac{1}{Z_{\rm side}} + 1\right|\right] = 20\log_{10}\left[\left|1 + \frac{\rho c}{2S_{\rm u}Z_{\rm side}}\right|\right] \\ &= 20\log_{10}\left[\left|1 + {\rm j}\frac{S}{2S_{\rm u}}\tan(k\ell_e)\right|\right] = 10\log_{10}\left[1 + \left(\frac{S}{2S_{\rm u}}\tan(k\ell_e)\right)^2\right] \end{aligned}$$

which is identical to Equation (8.82) in the textbook.

Another point to mention is that damping is not included in these analyses, and so the theoretical estimates of TL at the tuned frequency of the QWT, $f_{\rm QWT}$, will be infinite! Damping reduces the TL at the tuning frequency and increases it at other frequencies. Damping can be included in the impedance of the resonator in Equation (8.2) above by including a resistance term to account for losses at the entrance to the tube and losses due to oscillations along the length of the tube using Equation (8.33) in the textbook. Losses at the tube entrance depend, in practice, on the ratio of the cross-sectional areas of the main duct to resonator duct, the geometry of the throat, and flow speeds. For further information, see Section 8.8.2.2 and Howard and Craig (2014).

Example 8.2

Consider the double-tuned expansion chamber muffler shown in Figure 8.21(a) in the textbook, where the overall length of the muffler is $L = L_a + L_b + L_c = 0.5$ m, the acoustic lengths of the extension tubes are $L_b = L/2$ and $L_a = L_c = L/4$, and $d_{\rm ext} = 0.05$ m is the diameter of the extension tube, $D_{\rm exp} = 0.20$ m is the diameter of the expansion chamber, and the wall thickness of the extension tubes is $t_w = 0.001$ m. Calculate the TL of the muffler.

Solution 8.2

The 4-pole transmission matrices for the three acoustic elements of the DTEC muffler are shown in Equations (8.156) to (8.158) in the textbook. The total transmission matrix is calculated using Equation (8.159) in the textbook, and then it is used in Equation (8.142) in the textbook to calculate the TL. Figure 8.1 on the next page shows the predicted TL using the 4-pole theory of the DTEC muffler and an equivalent simple expansion chamber muffler (SEC) that has the same overall dimensions as the DTEC muffler. The results show that the DTEC muffler does not exhibit the unwanted dips in TL at the $1\times, 2\times, 3\times$ multiples of the normalised frequency, kL/π, which is a characteristic of an SEC muffler.

The TL of the DTEC muffler can also be predicted using finite element analysis. The geometric lengths of the extension tube segments must be shortened from their required acoustic lengths by the end corrections. Substitution of the dimensions into Equation (8.160) in the textbook, results in an end-correction length of $\ell_0 = 13$ mm. Hence the geometric length of the upstream extension tube should be $L_a = 500/4 - \ell_0 = 112$ mm, the middle straight duct section should be $L_c = 500/4 + 2\ell_0 = 151$ mm and the length of the downstream extension tube should be $L_b = 500/2 - \ell_0 = 237$ mm, so that the total length of the expansion chamber remains as $L = 112 + 151 + 237 = 500$ mm. The TL of the DTEC muffler calculated using finite element

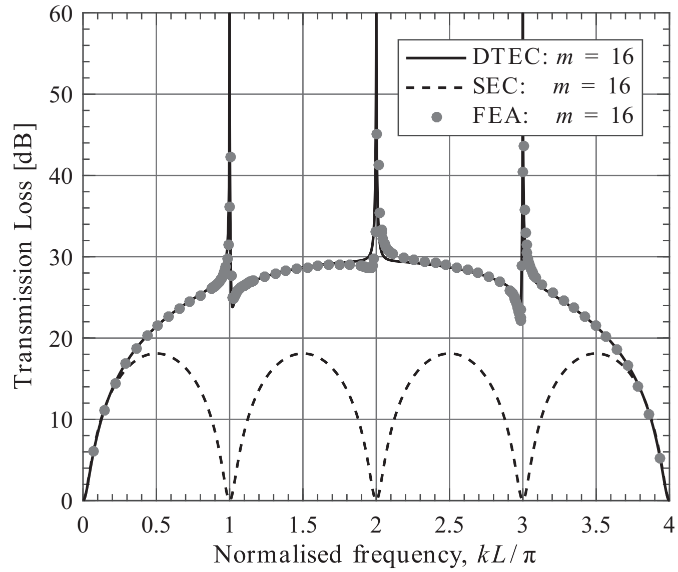


FIGURE 8.1 Transmission loss of a double-tuned expansion chamber muffler (DTEC), for Example 8.2, with the lengths of the extension tubes adjusted to account for the end corrections, a simple expansion chamber muffler (SEC) and predictions using finite element analysis (FEA) for the DTEC, with damping neglected.

analysis is shown in Figure 8.1, and closely matches the predictions using 4-pole transmission matrix theory.

Example 8.3

Ramya and Munjal (2014, Fig. 10) provide an example of a concentric tube resonator (CTR) with a perforated tube using a 1-D simulation method and a 3-D finite element analysis. The CTR muffler has a geometry as shown in Figure 8.23 in the textbook with dimensions and parameters listed in Table 8.2 on the next page. Calculate the TL of this CTR muffler.

Solution 8.3

A 1-D 4-pole transmission matrix model was created of the concentric tube resonator muffler using the theory in Section 8.9.9 in the textbook and implemented in the software package MATLAB in the script `TL_CTR_perforated.m` , which is available for download (MATLAB scripts for ENC, 2017). By using Equation (8.199) in the textbook, the length correction for extension tubes in the 4-pole model is $\ell_{0,1d} = 13.8$ mm. This value was used initially in the 4-pole model and adjusted until the TL spectra did not exhibit significant dips. The lengths of the upstream and downstream extension tubes were $L_{a,a} = L/2 - 0.0130 = 0.1875$ m, $L_{b,a} = L/4 - 0.0130 = 0.0873$ m. Figure 8.2 on the next page shows the predicted TL using the 4-pole model, finite element analysis simulation, and for a single expansion chamber (SEC) muffler of the same outer dimensions as the CTR muffler.

The TL of the CTR muffler was also evaluated using finite element analysis software (Ansys, release 17.1). By using Equation (8.203) in the textbook the length correction is $\ell_{0,3d} = 16$ mm. Hence the lengths of the upstream and downstream extension tubes in the finite element model were $L_{a,g} = (L/2) - \ell_{0,3d} = 0.1845$ m and $L_{b,g} = (L/4) - \ell_{0,3d} = 0.0843$ m, and the length of the perforated tube was $L_{c,g} = (L/4) + 2\ell_{0,3d} = 0.1322$ m. The impedance of the perforated tube

TABLE 8.2 Dimensions of the concentric tube resonator (CTR) muffler in Ramya and Munjal (2014), Fig. 10

Description	Parameter	Value
Main duct diameter	d_1	50.0 mm
Expansion chamber diameter	d_2	$3 \times d_1 = 150$ mm
Expansion chamber length	L	401 mm
Thickness of perforated tube	t_h	2 mm
Diameter of holes in perforated tube	d_h	3 mm
Percentage open area of perforated tube	P_{open}	27 %
Gas temperature	T	28.8 °C
Speed of sound	c	348.34 m/s
Density of gas	ρ	1.17 kg/m^3
Mach number	M_1, M_2	0
Dynamic viscosity of air	η	1.74×10^{-5} kg/ms
Length of upstream duct before muffler	L_{upstream}	200 mm
Length of downstream duct after muffler	$L_{\text{downstream}}$	200 mm

was determined by using Equation (8.163) in the textbook, and is used in the finite elements for the walls of the perforate tube. It can be seen in Figure 8.2 below that the estimate of the TL from the finite element analysis matches the prediction using the 4-pole transmission matrix theory, and matches the results published in Ramya and Munjal (2014, Fig. 10).

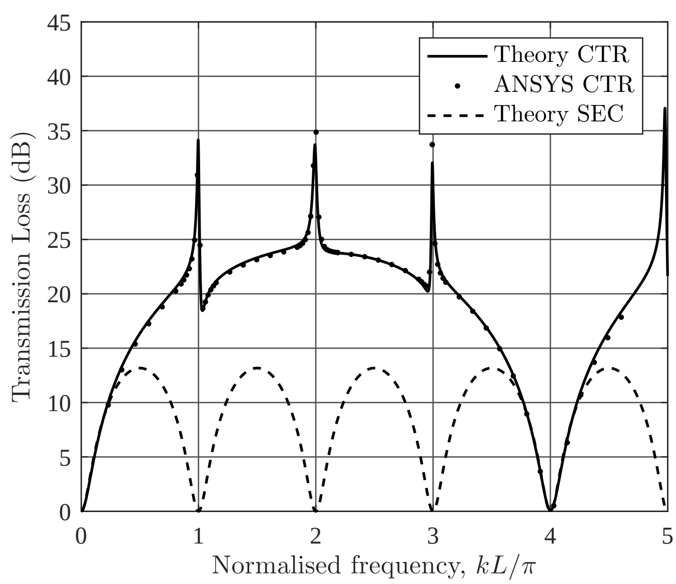


FIGURE 8.2 Transmission loss of the concentric tube resonator (CTR) comprising a perforated tube from (Ramya and Munjal, 2014, Fig. 10), calculated using a 4-pole transmission matrix model, finite element analysis (FEA), and a simple expansion chamber (SEC) muffler of equivalent dimensions calculated using Equation (8.155) in the textbook.

Example 8.4

Consider a circular duct that has a piston at one end, and a rigid termination at the other end. The parameters used in this example are listed in Table 8.3 below. Calculate the sound pressure along the duct for two cases: (1) when the gas in the duct has a constant temperature of 400°C, and (2) the gas has a linear temperature gradient between 20°C at the piston end and 400°C at the rigid termination.

TABLE 8.3 Parameters used in the analysis of a piston–rigid circular duct with a linear temperature gradient

Description	Parameter	Value	Units
Radius of duct	a	0.05	m
Length of duct	L	3.0	m
Velocity at rigid end	u_1	0.0	m/s
Velocity of piston	u_2	1.0	m/s
Temperature at rigid end	T_1	673	K
Temperature at piston end	T_2	293	K
Excitation frequency	f	200	Hz
Ratio of specific heats	γ	1.4	
Universal gas constant	R	8.3144621	J.mol^{-1} K^{-1}
Molar mass of air	M	0.029	kg.mol^{-1}
Atmospheric pressure	P_0	101325	Pa

Solution 8.4

The sound pressure along the duct can be calculated using the 4-pole transmission matrix theory by discretising the duct into short duct lengths and calculating the pressure and particle velocity at the ends of each segment.

The MATLAB script `temp_gradient_spl_along_duct_4pole_sujith.m`, which is available for download (MATLAB scripts for ENC, 2017), can be used to calculate the sound pressure and acoustic particle velocity in a duct with a temperature gradient, using the 4-pole transmission matrix method described in Section 8.9.10 in the textbook.

When attempting to analyse a system where there is a constant temperature profile in the duct, such that $T_1 = T_2$ and $g = 0$, then the parameter $\nu = 0$, as defined in Equation (8.214) in the textbook, causes numerical difficulties when evaluating the matrix elements T_{11} to T_{22} that contain $1/\nu$ terms. The work-around is to approximate a constant temperature profile by defining one end of the duct to have a small temperature offset, say 0.1°C, or replace the transmission matrix with Equation (8.137) in the textbook.

Figure 8.3 on the next page shows a comparison of the sound pressure level in the duct calculated using 4-pole transmission matrix theory, which was implemented in MATLAB and Ansys finite element analysis software for the cases where the temperature of the gas in the duct was at (1) an elevated temperature of 400°C, and (2) a linear temperature gradient of 400°C at the rigid end and 20°C at the piston end. The results from theoretical predictions agree with the results from finite element analysis. It can be seen that for the case of the temperature gradient along the duct, the distance between the first and second pressure nodes is smaller than the distance between the second and third pressure nodes. This is to be expected, as the gas temperature downstream is greater compared to upstream, which results in a higher sound speed and therefore a longer acoustic wavelength.

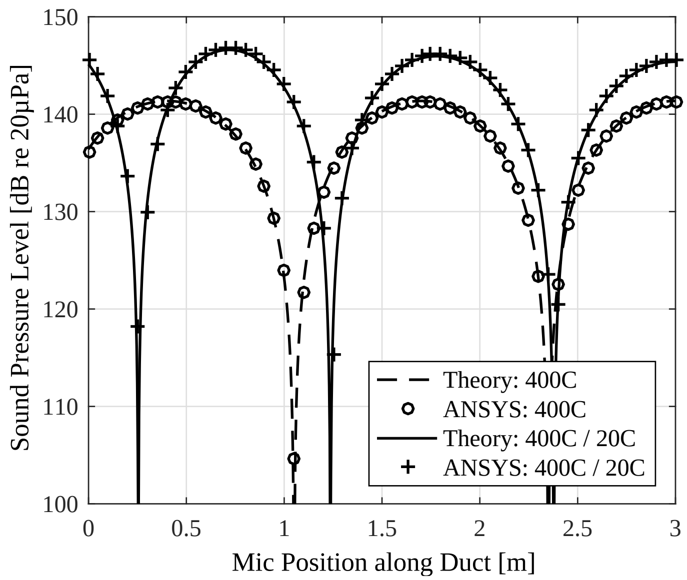


FIGURE 8.3 Sound pressure level versus axial location in a duct with a piston at one end and a rigid termination at the other, calculated using 4-pole theory and Ansys for the two cases of the gas having a linear temperature gradient from 400°C to 20°C, and for the gas having a constant temperature of 400°C along the length of the duct.

Example 8.5

The open section of a duct lined on two opposite sides is 0.2 m wide. Flow is negligible. What must be the length, thickness and liner flow resistance in the direction normal to the liner surface to achieve 15 dB attenuation at 100 Hz, assuming that there is no protective covering on the liner?

Solution 8.5

There will be many possible solutions. In practice, the question becomes, 'What is one acceptable solution?' For a liner with no solid partitions mounted normal to the liner surface to prevent axial wave propagation in the liner, the assumption of bulk-reaction is appropriate. It will also be assumed that the liner flow resistivity in the axial direction is the same as in the normal direction. Thus Figure 8.28 in the textbook is the appropriate figure to use for the liner attenuation calculations.

First, calculate the frequency parameter, taking the speed of sound as 343 m/s. That is, $2hf/c = 2 \times 0.1 \times 100/343 = 0.058$. Referring to Figure 8.28 in the textbook, one notes that a thick liner (curve 5) is implied according to the table associated with the figure. An alternative choice, to be investigated, might be curve 4, this choice being a compromise, promising a thinner liner. The best curve 5 of Figure 8.28 ($R_1\ell/\rho c = 4$) predicts an attenuation rate of 1.0 dB/h = 10.0 dB/m.

Next, Figure 8.36 in the textbook is used to calculate the required liner attenuation for a total TL (or IL if the attenuation of the muffler is greater than 5 dB) of 10 dB. Calculate the area ratio and value of kL to enter the figure. Assume for now that the final liner length will be 1 m. If this guess is wrong, then it is necessary to iterate until a correct solution is obtained:

$m = S_1/S_2 = 1 + \ell/h = 5$; and $kL = 2\pi fL/343 = 2\pi \times 100 \times 1.0/343 = 1.82$
where S_1/S_2 is the ratio of total duct cross-sectional area to open duct cross-sectional area.

From the figure, it can be seen that for an overall attenuation of 12 dB (TL), the liner attenuation must be approximately 7 dB (as the value of $kL = 1.82$ is close to the dashed curve value). A duct 1 m long will give an attenuation of 10.0 dB which is a bit high. Iterating finally gives a required duct length of 0.6 m corresponding to a liner attenuation of 6.0 dB and an overall attenuation (TL) of 13 dB.

Alternatively, curve 4 of Figure 8.28 in the textbook, predicts an attenuation rate as follows: Attenuation rate $= 0.6$ dB$/h = 6.0$ dB/m. Assuming a liner length of 1.6 m, calculate the area ratio and value of kL to enter Figure 8.36:
$m = S_1/S_2 = 1 + \ell/h = 3$; and $kL = 2\pi \times 100 \times 1.6/343 = 2.92$
From the figure, it can be seen that for an overall attenuation (TL) of 12 dB, the liner attenuation must be approximately 9.5 dB. A duct 1.6 m long will give an attenuation of 9.6 dB, which is the required amount.

In summary, use of curve 5 gives a duct 0.6 m long and $2(0.1 + 0.4) = 1.0$ m wide, whereas curve 4 gives a duct 1.6 m long and $2(0.1 + 0.2) = 0.6$ m wide. The required material flow resistance is such that $R_1\ell/\rho c = 4.0$.

9

Examples and Solutions, Chapter 9

Example 9.1

A machine is mounted on 4 identical vibration isolators. Calculate all the resonance frequencies that must be considered as the machine starts up and runs to full speed of 3000 RPM. Referring to figure 9.4 in the textbook, $2h = 0.1$ m, $2b = e = 2$ m, $a = 0.35$ m, $H = 4.2$ m, $D = 2.2$ m and the individual spring stiffnesses are:
$k_{sx} = k_{sz} = 1 \times 10^6$ N/m and $k_{sy} = 3 \times 10^6$ N/m. The mass, m, of the machine, supported on the 4 isolators is 1000 kg. The mass of each isolator is 3 kg. You may assume that the machine can be represented as a parallelepiped of uniform density.

Solution 9.1

There are six resonance frequencies that should be considered. They are associated with two rocking modes in each of the two orthogonal vertical planes, one rotational mode about the y-axis in Figure 9.4b in the textbook and one vertical mode.
The resonance frequency associated with vertical motion is given by Equation (9.12) in the textbook. Thus:
$f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{4 \times 3 \times 10^6}{1000 + 12/3}} = 17.4$ Hz.
The resonance frequency associated with the rotational mode about the y-axis is given by Equation (9.39) in the textbook. The moment of inertia of the machine, represented as a parallelepiped of uniform density, is given by:
$I_y = m((H)^2 + (D)^2)/12 = 1000(4.84 + 17.64)/12 = 1873$ kg m^2.
From Equation (9.39) in the textbook, the rotational resonance frequency is:
$f_y = (1/\pi)\sqrt{(10^6 + 4 \times 10^6)/1873} = 16.4$ Hz.
Now we will calculate the rocking (f_a) and horizontal (f_b) resonance frequencies for the vertical $x - y$ plane containing the dimension, $D = 2.2$ m.
The radius of gyration, δ_z, for rotation about the z-axis is:
$\delta_z = \sqrt{(4(a-h)^2 + D^2)/12} = \sqrt{(0.6^2 + 2.2^2)/12} = 0.658$ m.
From Equation (9.33) in the textbook, $W = (0.66/1)\sqrt{1/3} = 0.38$
From Equation (9.34) in the textbook, $M = 0.35/0.66 = 0.53$
From Equation (9.38) in the textbook,
$\Omega^2 = 0.5\left[0.38^2(1 + 0.53^2) + 1 \pm \sqrt{[0.38^2(1 + 0.53^2) + 1]^2 - 4 \times 0.38^2}\right] = 1.05$ and 0.138.
Thus, $\Omega = 1.02$ and $\Omega = 0.371$
Thus, $f_a = 0.371 \times 17.4/0.658 = 9.8$ Hz and $f_b = 1.02 \times 17.4/0.658 = 27.0$ Hz.

Now we will calculate the rocking (f_a) and horizontal (f_b) resonance frequencies for the vertical $z - y$ plane containing the dimension, $H = 4.2$ m.
The radius of gyration, δ_z, for rotation about the x-axis is:
$\delta_x = \sqrt{(4(a-h)^2 + H^2)/12} = \sqrt{(0.6^2 + 4.2^2)/12} = 1.22$ m.
From Equation (9.33) in the textbook, $W = (1.22/2)\sqrt{1/3} = 0.352$
From Equation (9.34) in the textbook, $M = 0.35/1.22 = 0.287$
From Equation (9.38) in the textbook,
$\Omega^2 = 0.5\left[0.38^2(1+0.53^2) + 1 \pm \sqrt{[0.38^2(1+0.53^2)+1]^2 - 4 \times 0.38^2}\right] = 1.01$ and 0.124.
Thus, $\Omega = 1.01$ and $\Omega = 0.352$
Thus, $f_a = 0.352 \times 2 \times 17.4/1.22 = 10.0$ Hz and $f_b = 1.01 \times 2 \times 17.4/1.22 = 28.2$ Hz.
Thus the resonance frequencies are:

- $f_0 = 17.4$ Hz
- $f_y = 16.4$ Hz
- $f_{a1} = 9.8$ Hz
- $f_{b1} = 27.0$ Hz
- $f_{a2} = 10.0$ Hz
- $f_{b2} = 28.2$ Hz

The rotational speed is 50 Hz (3000/60) so this should be OK, although two of the coupled rocking and horizontal resonance frequencies may be a bit high if a substantial amount of horizontal vibration isolation is required at 50 Hz ($f/f_0 \approx 1.8$ – see Figure 9.2 in the textbook).

Example 9.2

A machine of mass 800 kg is mounted on isolators, which in turn are mounted on the first floor of a building. The machine rotational speed is 3000 RPM and when tested with its vibration isolators mounted on a rigid concrete floor directly on top of the ground, the resonance frequency was 20 Hz. The dynamic stiffness of the floor structure can be approximated as $k_{sf} = 2 \times 10^6\omega$ N/m and the dynamic mass of the floor structure can be approximated as $800/\omega$ in the frequency range of interest. The machine is supported on 4 vibration isolators of total mass 5 kg.

(a) What is the vertical stiffness (in N/m) of each isolator in the single stage isolation system?
(b) What would be the reduction in force transmission to a rigid foundation (in dB) as a result of the vibration isolators at the machine operating speed if the critical damping ratio of the isolators were 0.05?
(c) What is the reduction in vibration level (dB) of the first floor at the machine operating speed, as a result of using the vibration isolators?
(d) Using the same total isolator mobility as in the single stage isolation system, what would be the maximum possible improvement in vibration isolation (in dB) at the machine operating speed if a two-stage isolation system were used with an intermediate mass of 200 kg?

Solution 9.2

(a) The total vertical stiffness may be calculated using Equation (9.12) in the textbook. Thus:
$k_s = (2\pi \times 20)^2 \times (800 + 5/3) = 12.66$ MN/m

(b) The reduction in transmitted force in decibels $= -10\log_{10} T_F$, where T_F can be calculated using Eq. (9.26c) in the textbook, where $X = f/f_0 = (3000/50)/20 = 3$.
Thus: dB reduction $= -5\log_{10} \dfrac{1 + (2 \times 0.05 \times 3)^2}{(1-9)^2 + (2 \times 0.05 \times 3)^2} = 8.8$ dB.

(c)

(d)

Example 9.3

Our objective is to design an optimal vibration absorber for an item of rotating machinery to prevent it being damaged as it runs up to speed after being shut down. The problem speed at which vibration levels are at a maximum for the machine on its vibration isolation system during run-up is is 1500 RPM. Currently, the vibration is being limited by snubbers but these are having to be replaced after every start up. The machine has mass of 1000 kg and is mounted on a spring isolation system characterised by a spring mass of 18 kg and no significant damping. The maximum allowed added mass (excluding springs and dampers) is 100 kg.

(a) What is the optimal stiffness for the spring part of the absorber?

(b) What is the optimal critical damping ratio of the damper part of the absorber?

(c) What is the approximate maximum vibration amplitude of the machine that can be expected after installation of the absorber if the out-of-balance in the machine is caused by a mass of 10 kg attached to the rotating shaft of the machine at a radius of 20 mm from the central axis of rotation?

Solution 9.3

(a) The maximum vibration amplitude occurs at the primary system resonance frequency, which is $f_0 = 1500/60 = 25$ Hz.
For a mass, m_1 of 1000 kg supported on an isolation system with a resonance frequency of $f_0 = 25$ Hz, the effective stiffness of the isolation system can be found by rearranging Equation (9.12) in the textbook to give:
$k_{s1} = (2\pi f_0)^2 \times (m + m_{1s}/3) = (2\pi \times 25)^2 \times (1000 + (18/3)) = 24.8 \times 10^6$ N/m.
From equation (9.77) in the textbook, the required optimal absorber stiffness is:
$k_{s2} = 24.8 \times 10^6 \times (100/1000)/(1 + 100/1000)^2 = 2.05 \times 10^6$ N/m.

(b) The optimum critical damping ratio is given by equation (9.78) (corrected according the the errata) in the textbook. Thus:
$\zeta_2^2 = 3 \times 0.1/(8 \times (1 + 0.1)^3) = 1.609 \times 10^{-2}$.
Thus, the optimum critical damping ratio, $\zeta_2 = 0.13$.

(c) The out-of-balance force amplitude is given by, $|F| = mr\omega^2$ (basic dynamics).
Thus, $|F| = 10 \times 0.02 \times (2\pi \times 25)^2 = 4934\ N$.
From Figure 9.14 in the textbook, $|y_1|k_{s1}/|F| \approx 4.5$
Thus, the maximum vibration amplitude of the machine after installation of the absorber is:
$|y_1| = 4.5 \times 4934/(24.8 \times 10^6) = 0.90$ mm

10

Examples and Solutions, Chapter 10

Example 10.1

Estimate the interior sound power level spectrum for an axial compressor of 15 blades and 80 kW power turning at 3000 rev min^{-1}.

Solution 10.1

1. Use Equation (10.8) to calculate f_p:

$$f_p = 15(3000)/30 = 1500 \text{ Hz}$$

2. Use Equation (10.13) to calculate f_h:

$$f_h = 1500^2/400 = 5600 \text{ Hz}$$

3. Use Equations (10.9)–(10.11) to calculate power level in bands:

For the 63 Hz octave band:

$$L_W = 76.5 + 10\log_{10} 80 = 95.5 \text{ dB}$$

For the 500 Hz octave band:

$$L_W = 72 + 13.5\log_{10} 80 = 97.7 \text{ dB}$$

f_p lies in the 2000 Hz octave band (see Table 1.2 in the textbook):

$$L_W = 66.5 + 20\log_{10} 80 = 104.6 \text{ dB}$$

f_h lies in the 4000 Hz octave band (see Table 1.2 in the textbook):

$$L_W = 72 + 13.5\log_{10} 80 = 97.7 \text{ dB}$$

Using the values calculated above, follow the procedure outlined in the text to sketch the estimated spectrum (L_W vs $\log_{10} f$).

Example 10.2

Gas is to be exhausted under pressure from a 100 mm diameter pipe to the atmosphere at a speed of 230 m/s. The density of the gas is 1.5 kg/m^3 and its temperature is essentially that of the ambient atmosphere into which it is to be exhausted. Determine the overall sound pressure levels at 10 m from the jet orifice and at various angles relative to the direction of the jet.

Solution 10.2

The speed of sound has been taken as 343 m/s in the air surrounding the jet and the jet temperature, T is equal to the ambient temperature, T_0. The density of the atmosphere is $\rho_0 = 1.206$ kg/m^3.

1. Determine the jet efficiency using Equation (10.25) in the textbook.

$$\eta = (T/T_0)^2(\rho/\rho_0)K_a M^5 = 1.0 \times (1.5/1.206) \times 5 \times 10^{-5} \times (230/343)^5 = 8.43 \times 10^{-6}$$

where K_a is given in the textbook as 5×10^{-5}.

2. Calculate the mechanical stream power using Equation (10.24):

$$W_m = \rho U^3 \pi d^2/8 = 1.5 \times 230^3 \times \pi \times (0.1)^2/8 = 71.7 \qquad \text{(kW)}$$

3. Calculate the overall acoustic power using Equation (10.23):

$$W_a = \eta W_m = 8.43 \times 10^{-6} \times 71700 = 0.60 \qquad \text{(W)}$$

4. Calculate the overall sound power level using Equation (10.26):

$$L_W = 10 \log_{10} W_a + 120 = 117.8 \qquad \text{(dB re } 10^{-12} \text{ W)}$$

5. Use Equation (10.27) to determine the overall sound pressure level at 10 m from the jet orifice:

$$L_p = L_W + \text{DI} - 10 \log_{10}(4\pi r^2) = \text{DI} + 86.8 \qquad \text{(dB re } 20\,\mu\text{Pa)}$$

6. Use Table 10.2 in the textbook and the equation in item (5) above to construct the following table:

Sound pressure level versus angle from jet axis for Example 10.2

Angle from jet axis (degrees)	Overall sound pressure level (dB re 20 µPa)
0	87
20	88
40	95
60	89
80	83
100	79
120	76
140	74
160	72
180	70

7. Determine the octave band of maximum sound pressure level. First, calculate the spectrum peak frequency, f_p, using the Strouhal number, $N_s = 0.2$ and Equation (10.28):

$$f_p = 0.2U/d \text{ (Hz)} \; = 0.2 \times 230/0.1 = 460 \text{ Hz}$$

Reference to Table 1.2 in the textbook shows that this frequency lies in the 500 Hz octave band.

8. Use Figure 10.3 in the textbook and the above information to construct the following table.

Octave band sound pressure level versus angle from the jet axis, for Example 10.2

Angular position relative to the direction of the jet (degrees)	Octave band centre frequency (Hz)							
	63	125	250	500	1000	2000	4000	8000
	Sound pressure level (dB re 20 µPa)							
0	72	77	80	82	80	76	69	63
60	74	79	83	84	82	78	71	65
120	61	66	70	71	69	65	58	52
180	55	60	64	65	63	59	52	46

11

Examples and Solutions, Chapter 11

Example 11.1

Consider a rigid-walled box with dimensions 2.5 m $\times$ 3 m $\times$ 5 m. In one corner of the box at $(0.5, 0.5, 0.5)$ is a monopole source radiating sound that has a mass volume velocity (i.e., density times volume velocity) amplitude of 0.01 kg/s. Assume that box is filled with a gas that has a speed of sound of 343 m/s, and a density of $\rho = 1.29$ kg/m^3. Determine the sound pressure level versus frequency at a point $(2.0, 1.5, 2.5)$, and plot the sound pressure level distribution on the walls of the box at the second resonance frequency.

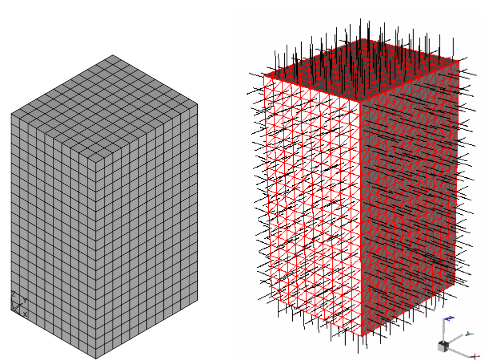
Solution 11.1

A boundary element analysis will be conducted using the FastBEM software. The initial step is to create the boundary element mesh of the box. An Ansys script is shown below that can be used to create a box, and mesh the surfaces of the box with (rectangular) quadrilateral `SHELL63` elements. The model is then exported as an Ansys archive file that has a filename extension `.cdb`. The model created in the Ansys software comprises rectangular shaped elements. However, when the mesh is imported into FastBEM, each rectangular element on the surface of the box is divided into 2 triangular elements. Note that the rule of thumb is that there should be at least 6 elements per wavelength (Marburg, 2002). For an analysis at 200 Hz, the elements should be smaller than $c/(6f) = 343/1200 = 0.258$ m, and so for this example, the nominal element size selected is 0.25 m.

Ansys APDL commands to create a block, mesh the block with `SHELL63` elements, and export the model.

```
/PREP7
ET,1,SHELL63
BLOCK,0,2.5,0,3,0,5,
/PSYMB,ADIR,1
ESIZE,0.25,0,
AMESH,ALL
TYPE,1
REAL,1
MAT,1
NUMMRG,NODE
NUMMRG,ELEM
CDWRITE,DB,'ANSYS_Model','cdb',,'',''
```

Figure 11.1(a) shows the mesh of the box that was created in Ansys using the script.



(a) (b)

FIGURE 11.1 (a) Ansys model comprising `SHELL63` elements on the faces of the box to be imported into the FastBEM software. (b) FastBEM model of the box showing that the normal for each element is pointing away from the acoustic domain.

Start the FastBEM software and click on File > Import Model > From ANSYS Archive. Select the file `ANSYS_Model.cdb` and then click the Import button. The next steps involve confirming that the model was imported correctly, and then configuring the analysis.

By left-clicking the mouse button and holding it down, it is possible to rotate the model to inspect each face of the box. Click on the menu on the top of the screen Plot Control > Normal Vector. In the dialog box that appears, click in the white square next to Display so that a tick appears, and then click the OK button. The model of the box should have 'spikes' to indicate that the normal vector for each element is pointing away from the box, as shown in Figure 11.1(b). Rotate the view of the model to inspect each face of the box and check that the arrows (spikes) are pointing away from the box.

It is a requirement that the normals of each element are orientated in the correct direction to indicate the region of the acoustic domain. For this problem, where there is an enclosed acoustic domain (also called an interior problem), the element normals must be pointing outwards from the acoustic domain. When creating a model using Ansys, before meshing the solid model comprising of areas, one should check that the normals for each *area* are orientated correctly. For this example, which used the script listed previously, the normals should be pointing outwards from the rectangular box.

The next steps involve configuring the analysis parameters in FastBEM. In the menu tree on the left side of the screen, under the branch 'Parameters', click 'Job Options'. In the dialog box that appears, click the tab, 'Solver Type', click the button next to 'Conventional BEM', and then click the 'OK' button.

Next, an acoustic monopole source will be defined at the coordinate $(0.5, 0.5, 0.5)$, which has a mass volume velocity of 0.01 kg/s. In the menu on the left side of the screen under the branch, 'Parameters', click 'Acoustic Sources'. In the dialog box that appears, click the tab, 'Monopoles'. In the text entry box next to 'No. of Monopoles', enter the number, '`1`', and then click the icon of the two green arrows, which will refresh the number of rows in the entry table, so that there should be a row with the number, '`1`', in the column labelled, 'NO'. In the cell beneath the text, 'Complex amplitude', enter the number, `0.01`, in the left cell, and `0.0` in the right cell. In the three cells beneath the label, 'Location Vector', in each of the three cells, enter the value, '`0.5`', to define the coordinate of the monopole source at $(0.5, 0.5, 0.5)$. Figure 11.2 below shows the dialog box with the corresponding entries. Click the 'OK' button when finished.

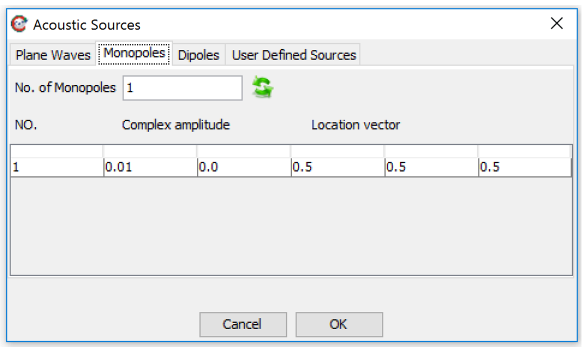


FIGURE 11.2 Dialog box for defining the location and amplitude of the acoustic monopole source in FastBEM.

The next step is to define the material properties for the gas in the box. In the menu on the left side of the screen under the 'Parameters' branch click Material. In the dialog box that appears, enter the values for 'Sound Speed' as '`343.0`', 'Mass Density' as '`1.29`', 'Ref. Pressure', '`2.0E-5`', 'Ref. Intensity `1.0E-12`', and the 'Complex Wavenumber k Ratio' as '`0`'. Click the 'OK' button when finished.

The next step is to define the analysis frequency range. For this example, the analysis will be conducted from $f_1 = 10$ Hz to $f_2 = 200$ Hz, in frequency increments of $\Delta f = 2.5$ Hz. In the menu on the left side of the screen under the branch, 'Parameters', click on the item, 'Frequency'. In the entry box, 'Freq. 1', type '`10.0`' and in the box next to 'Freq. 2', type '`200.0`'. The (integer) number to enter into the cell next to the label, 'No. of Freq', can be calculated as:

$$N = \left\lfloor \frac{f_2 - f_1}{\Delta f} \right\rfloor + 1 = \left\lfloor \frac{200 - 10}{2.5} \right\rfloor + 1 = 77 \tag{11.1}$$

In the cell next to 'No. of Freq', type '`77`'. Click the 'OK' button when finished.

In this example, the sound pressure level at the point $(2.0, 1.5, 2.5)$ will be calculated. By default, the FastBEM software will calculate the sound pressure on the boundary surface of the model. If results at other locations are required, such as at points inside the acoustic domain, then a mesh of nodes and elements must be defined at the required location, which are called

'Field Points' and 'Field Cells' in the FastBEM software. Other BEA software packages call this a '*data recovery mesh*'. The 'Field Points' and 'Field Cells' that have a complicated geometry can be defined using Ansys. However for this example, the FastBEM interface will be used. In the menu on the left side of the screen under the branch, 'Solution', click 'New Field Surface'. In this example, the sound pressure level at a point is required, so a small surface will be defined for the 'Field Points' and 'Field Cell'. The surface will have dimensions, 0.01 m × 0.01 m, which is much smaller than the acoustic wavelength at 200 Hz. In the entry boxes for 'Length' and 'Height', type '`0.01`'. In the entry boxes for 'Length Partition' and 'Height Partition', type '`1`'. In the drop-down menu for 'Plane', select the option, '`YZ`'. In the entry box for 'Offset Distance', type '`0.75`'. A rectangular mesh will be defined in the YZ-plane, which is offset from the centroid of the model along the global x-axis defined by the value for 'Offset Distance'. In this example, the desired measurement location is at $x = 2.0$ m, the size of the model along the x-axis is 2.5 m, and hence the required offset distance from the centroid of the model for the plane of 'Field Points' is $2.5/2 - (2.5 - 2.0) = 0.75$ m. Figure 11.3 shows the dialog box entries to define the 'Rectangular mesh' for the 'Field Points' and 'Field Cells'. Click the 'OK' button when finished.

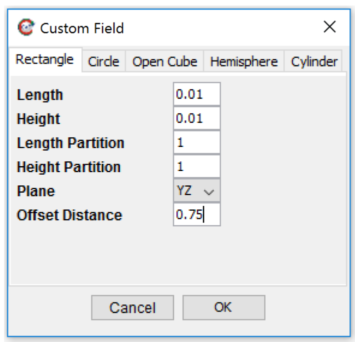


FIGURE 11.3 Define the mesh for the Field Point and Field Cells.

The next step is to solve the model. Save the project by clicking 'File > Save Project As', and type an appropriate filename. In the menu on the left side of the screen under the branch, 'Solution', click 'Start Solver'. In the dialog box that appears there are a number of tabs that can be clicked to verify that the options defined in the previous steps have been entered correctly. Once these have been confirmed, click the 'Start' button. A dialog box will appear requesting the user to enter a new filename and folder to store the results. Enter an appropriate filename such as '`BEM_output`' and then click the 'Start' button. A dialog box will appear with the title, 'Job Running', and will show the progress of the calculations.

Once the calculations have completed, in the menu on the left side of the screen under the branch, 'Plot FR Curve', click 'SPL_max(dB)' and a graph will be displayed showing the average sound pressure level over the 'Field Cells' versus frequency.

Figure 11.4 shows the predicted sound pressure level using the FastBEM software, using modal summation theory described in Nefske and Sung (2006), Sec. 6.7, p. 161, which was implemented in MATLAB, and Ansys finite element analysis software. The MATLAB script, '`rect_cav_3D.m`', used to calculate the sound pressure level inside the rigid-walled cavity is available for download from MATLAB scripts for ENC (2017). The 'Ansys Mechanical APDL' script used to calculate the sound pressure inside the cavity, '`rigid_cavity_full.inp`', can also be downloaded from MATLAB scripts for ENC (2017). The results show that there is good agreement between all three analysis methods. At frequencies above 110 Hz, it can be seen that the results for the prediction methods differ slightly, which is, in part, due to using only 6 elements per wavelength. Using a mesh density with a greater number of elements per wavelength would improve the agreement between the prediction methods.

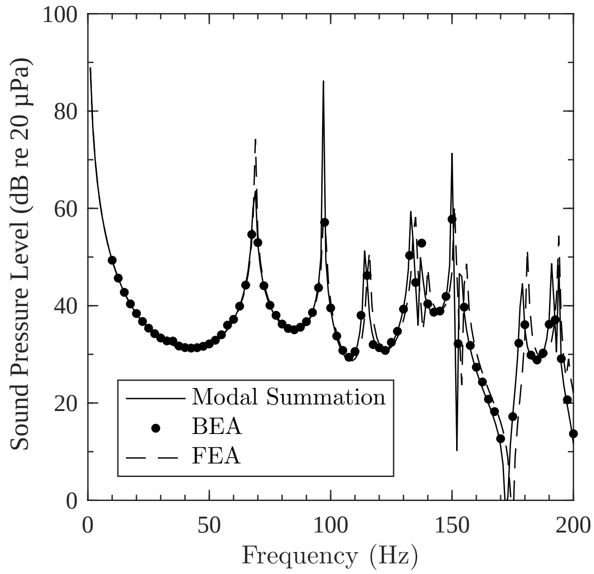


FIGURE 11.4 Sound pressure level at the point $(2.0, 1.5, 2.5)$ calculated using modal summation theory, boundary element analysis (BEA) and finite element analysis (FEA).

It can be seen in Figure 11.4 above that there is peak in the sound pressure level response at 97.5 Hz, which is likely to be the second acoustic resonance of the box. The sound pressure level on the surface of the box at this frequency can be inspected by selecting the appropriate results set, which in this case is set number $(97.5 - 10)/2.5 + 1 = 36$. In the menus on the right side of the screen, make sure there is a tick in the white square next to 'Contour Plot'. In the menu region labelled, 'Frequency', click the downwards pointing arrow to reveal the calculation sets, and select the value, '`36`'. The status line at the bottom of the screen should display, '`Freq. No. = 36, f= 0.9750D+02 (Hz)`', indicating that the results at 97.5 Hz will be displayed. In the menu on the left side of the screen under the branch, 'PostProcessor', click 'Plot SPL (dB)' to display the sound pressure level at 97.5 Hz. The number and range of contours can be changed by clicking 'Setting > Color Display Setting', selecting the 'Continuous' option and changing the lower and upper bounds to '`20.0` and `70.0`', respectively. Figure 11.5 shows the contour plot of the sound pressure levels at 97.5 Hz on the interior surface of the volume. The FastBEM software can export the results into a file format that can be imported into the software, 'Tecplot' (Tecplot Software, 2017), which enables the creation of presentable figures, or imported into MATLAB for further post-processing.

The contour plot of the sound pressure levels suggests there is 1 pressure node (minimum) along the x-axis, 0 nodes along the y-axis and 2 nodes along the z-axis. The resonance frequencies of a rigid-walled rectangular volume can be calculated using Equation (**??**), and after substituting the modal indices and other parameters gives:

$$
\begin{aligned}
f &= \frac{343}{2}\sqrt{\left[\frac{1}{2.5}\right]^2 + \left[\frac{0}{3}\right]^2 + \left[\frac{2}{5}\right]^2} \\
&= 97.0 \text{ Hz}
\end{aligned}
$$

Hence, the second peak in the sound pressure level spectra at 97.5 Hz, as shown in Figure 11.4, corresponds to the $(1, 0, 2)$ mode of the rectangular enclosure, which has a resonance frequency of 97.0 Hz.

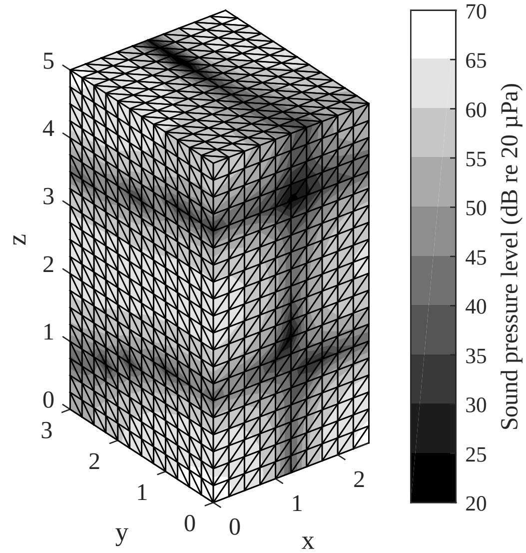


FIGURE 11.5 Sound pressure level on the interior surface of the box at 97.5 Hz calculated using the FastBEM software.

Example 11.2

Consider a rigid-walled rectangular enclosure containing air with dimensions $0.5 \times 0.3 \times 1.1$ m as shown in Figure 11.6. A simply supported, flexible panel is attached at one end of the enclosure with dimensions 0.5×0.3 m. The flexible panel is 3 mm thick aluminium with a Young's modulus of $E = 70.9$ GPa and a density of 2700 kg/m^3. A point force of $F = 1$ N acts on the panel at $(0.10, 0.06, 0.0)$, which causes the panel to vibrate and radiate sound into the enclosure. Calculate the sound pressure level inside the enclosure at location $(0.300, 0.105, 0.715)$.

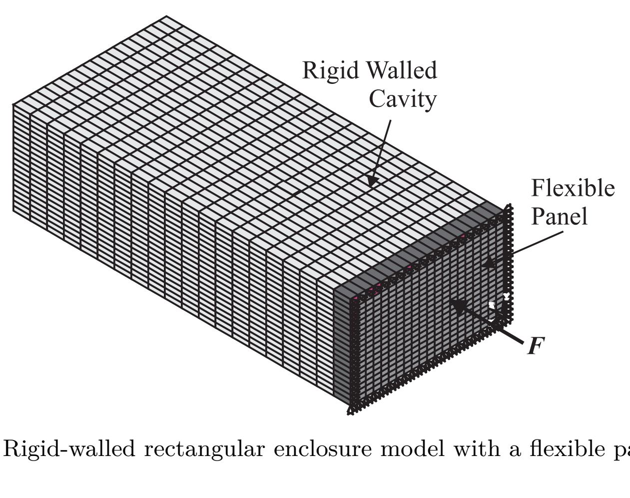


FIGURE 11.6 Rigid-walled rectangular enclosure model with a flexible panel on one end.

Solution 11.2

The problem was analysed using the numerical method described above and implemented in the software MATLAB. An *in vacuo* (meaning without the air inside the enclosure) modal analysis of the flexible panel was conducted and 21 mode shapes and resonance frequencies were calculated. Similarly, the modal response of the enclosure volume was calculated, where it was modelled as a rectangular volume enclosed by rigid walls, resulting in 102 acoustic resonance frequencies and pressure mode shapes. The range of the resonance frequencies from the modal analyses covers two octaves higher than the frequency range of interest, and the distribution of resonance frequencies for the flexible panel and the acoustic cavity are shown in Figure 11.7.

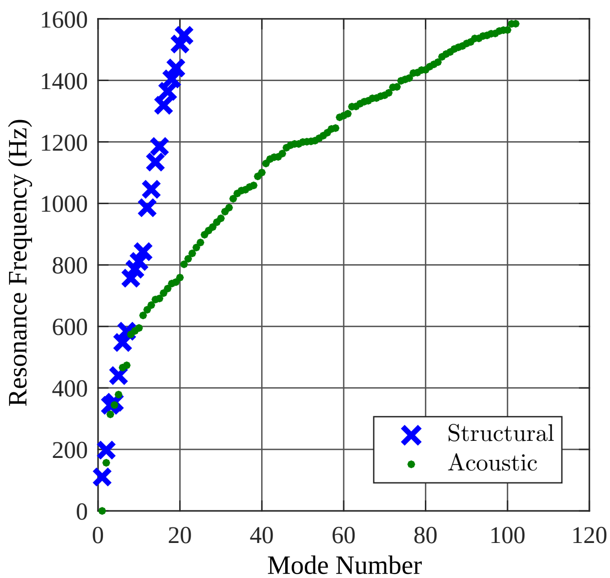


FIGURE 11.7 Structural and acoustic resonance frequencies.

The problem was also analysed using the finite element analysis software ANSYS. Pressure formulated acoustic elements (FLUID30) were used to model the acoustic cavity, and shell elements (SHELL63) were used to model the thin aluminium plate. For the parts of the enclosure volume not in contact with the enclosing structure, acoustic elements with only a pressure degree of freedom were used. At the interface between the flexible panel and the acoustic elements, the acoustic elements had pressure and structural displacement degrees of freedom, to enable the coupling between the flexible panel and the acoustic fluid. The fluid–structure interaction option was enabled for the flexible panel elements in contact with the acoustic elements that have both pressure and translational DOFs. The number of element divisions used along each side of the enclosure was 20. At 1600 Hz the wavelength is $343/1600 = 0.214$ m, and the longest element length is $1.1/20 = 0.055$ m. Hence the ratio between the wavelength to largest element size is $0.214/0.055 = 3.9$, which is less than the recommended 6 elements per wavelength. However, in the frequency range of interest (400 Hz) the ratio of elements per wavelength is $[(343/400)/(1.1/20)] = 15.6$.

Figure 11.8 shows a comparison of the sound pressure response calculated for a point, (0.300, 0.105, 0.715), within the enclosure using FEA with a full fluid–structure interaction formulation, and the modal coupling method implemented using MATLAB. It can be seen that the results are almost identical over the frequency range of interest.

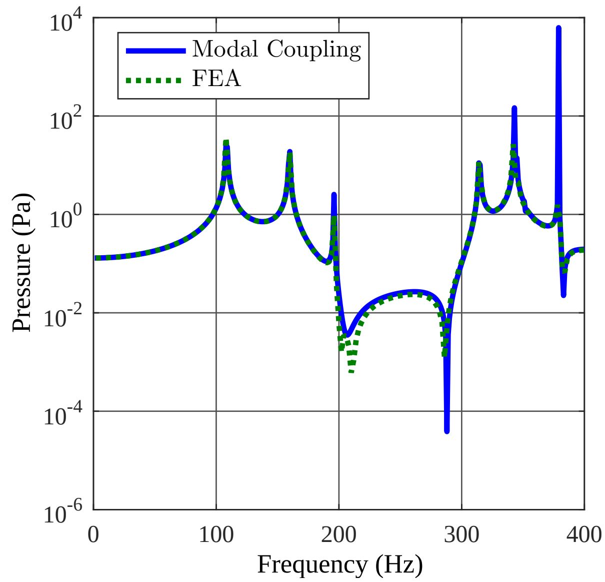


FIGURE 11.8 Pressure at a node, (0.300, 0.105, 0.715), inside a rectangular enclosure, evaluated using modal coupling theory and FEA (ANSYS) using full fluid–structure interaction coupling.

Example 11.3

Consider a system comprising a mild steel beam 2 m long, with a cross section of 20 mm $\times$ 20 mm, which is welded to the centre of a large mild steel plate of dimensions, 3 m $\times$ 3 m, with a thickness of 20 mm. The plate is mounted as a baffle in one wall of a room that has edge lengths of equal size of 5 m, hence a volume of 125 m^3, and the room has an average Sabine absorption coefficient of $\bar{\alpha} = 0.07$. A power of 1 W is injected into the beam, using a point moment at its end to generate flexural waves in the beam. The beam is point coupled to the plate, and in this case only generates bending waves in the plate. For the frequency range 63 Hz to 2 kHz, calculate the sound pressure level in the room. Assume that the damping loss factors for the beam and plate are both 0.15.

Solution 11.3

The general procedure for solving SEA problems is:

1. Identify the subsystems and their geometric and material parameters.
2. Sketch a network diagram of the overall system to identify the power flows between subsystems and the method of power transmission at each junction. For example, consider if power transmission occurs due to translational or rotational coupling, or both.
3. Calculate the input impedances for each subsystem.
4. Calculate the modal densities of the subsystems.
5. Calculate the coupling loss factors between the subsystems.
6. Calculate the damping loss factors for each subsystem.
7. Calculate the input powers to each subsystem.

8. Form the matrix equation that describes the power flows between subsystems, and calculate the energy levels within each subsystem.
9. Calculate the amplitude responses of the subsystems from the results of Step 8.

The MATLAB script called `book_sea_example.m` used to calculate the sound pressure level inside the rigid-walled cavity is available for download from MATLAB scripts for ENC (2017).

Step 1: Parameters
The parameters for the problem are contained in the problem description.

Using the values in Appendix C in the textbook for mild steel, $E = 207$ GPa, density $\rho = 7850$ kg/m^3 and Poisson's ratio $\nu = 0.3$, the longitudinal wave speed in a thin plate is given by Equation (7.5) in the textbook as:

$$c_{L\text{plate}} = \sqrt{\frac{E}{\rho_m(1-\nu^2)}} = \sqrt{\frac{207 \times 10^9}{7850\,(1-0.3^2)}} = 5383 \text{ m/s} \tag{11.2}$$

Step 2: Network Diagram
A system diagram can be drawn illustrating the power flow between subsystems as shown in Figure 11.9.

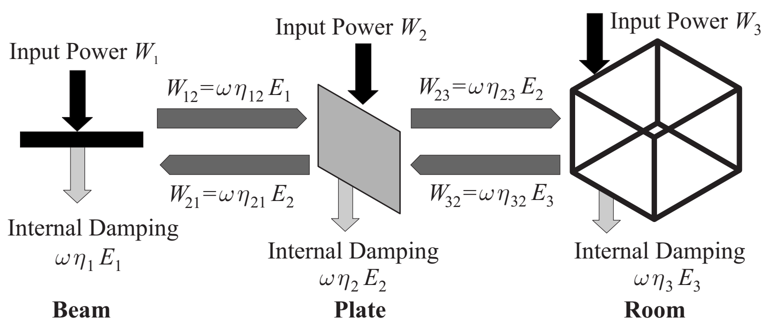


FIGURE 11.9 Power flow between subsystems.

Step 3: Impedances of subsystems
The impedances of the beam and plate can be calculated. From Table 11.1 in the textbook together with the definitions of k_B following Table 11.5 in the textbook and the relationship, $k_B = \omega/c_B$, the impedance of the beam for moment coupling is given by:

$$Z_{M\text{beam}}(\omega) = \frac{(1-\mathrm{j})\rho_m S c_B}{2k_B^2} = \frac{(1-\mathrm{j})\rho_m S\omega}{2k_B^3} = \frac{(1-\mathrm{j})\rho_m S c_B^3}{2\omega^2} \tag{11.3}$$

where

$$k_{B\text{beam}} = [\rho_m S_b \omega^2/(EJ_y)]^{1/4} = \left[\frac{7850 \times (0.02)^2\omega^2}{207 \times 10^9 \times (0.02^4/12)}\right]^{1/4} = 0.1837\omega^{1/2} \tag{11.4}$$

where

$$J_y = bh^3/12 = 0.02^4/12 = 1.333 \times 10^{-8} \tag{11.5}$$

Thus:

$$Z_{M\text{beam}}(\omega) = \frac{(1-\mathrm{j}) \times 7850 \times (0.02)^2 \times \omega}{2 \times 0.1837^3 \times \omega^{3/2}} = 253.4(1-\mathrm{j})\omega^{-1/2} \tag{11.6}$$

From Table 11.2 in the textbook, the impedance of a thin plate due to an applied moment is given by:

$$Z_{M\text{plate}}(\omega) = \frac{16\omega m}{k_B^4}\left[\frac{1}{1-\dfrac{4\mathrm{j}}{\pi}\log_\mathrm{e}(0.89k_Br)}\right] = \frac{16B}{\omega}\left[\frac{1}{1-\dfrac{4\mathrm{j}}{\pi}\log_\mathrm{e}(0.89k_Br)}\right] \tag{11.7}$$

where $r = 20\times10^{-3}/2$ m is half the thickness of the beam, $k_B = \omega/c_B$ and B is the plate bending stiffness given by Equations (7.1) and (7.2) in the textbook.

Rewriting in terms of real and imaginary parts:

$$Z_{M\text{plate}}(\omega) = \frac{16B}{\omega\,(1+[(4/\pi)\log_\mathrm{e}(0.89k_Br)]^2)}\left[1+\mathrm{j}(4/\pi)\log_\mathrm{e}(0.89k_Br)\right] \tag{11.8}$$

$$k_{B\text{plate}} = (\omega^2 m/B)^{1/4} = (\omega^2\rho h/B)^{1/4} = (\omega^2\times7850\times0.02/1.516\times10^5)^{1/4} = 0.179\omega^{1/2} \tag{11.9}$$

where for a plate of thickness, h, the bending stiffness, B, from Equation (7.2) in the textbook is:

$$B = Eh^3/[12(1-\nu^2)] = 207\times10^9\times0.02^3/[12(1-0.3)^2] = 1.516\times10^5 \tag{11.10}$$

Thus:

$$Z_{M\text{plate}}(\omega) = \frac{16\times1.516\times10^5}{\omega\{1+(4/\pi)\log_\mathrm{e}(0.89\times0.179\omega^{1/2}\times0.01)]^2\}}[1{+}\mathrm{j}(4/\pi)\log_\mathrm{e}(0.89{\times}0.179\omega^{1/2}{\times}0.01)] \tag{11.11}$$

Step 4: Modal Densities
The modal densities of the three subsystems are calculated using Tables 11.6. 11.7 and 11.8 in the textbook as:

Beam:

$$n_\text{beam}(\omega) = \frac{L_B}{2\pi}\left[\frac{\rho_m S}{EJ_y\omega^2}\right]^{1/4} = \frac{2}{2\pi}\left[\frac{7850\times0.02^2}{207\times10^9\times1.333\times10^{-8}\omega^2}\right]^{1/4} = 0.0585\omega^{-1/2} \tag{11.12}$$

Plate (independent of frequency):

$$\begin{aligned} n_\text{plate} &= (\sqrt{3})S/(2\pi hc_L) \\ &= (\sqrt{3})(3\times3)/(2\pi\times20\times10^{-3}\times5383) \\ &= 0.0230 \text{ modes per radian} \end{aligned} \tag{11.13}$$

Room:
The modal density for a room varies with frequency (see Table 11.8 in the textbook for an equation in terms of angular frequency, ω):

$$n_\text{room}(\omega) = \frac{V\omega^2}{2\pi^2c^3} + \frac{S\omega}{8\pi c^2} + \frac{L}{16\pi c} \tag{11.14}$$

where S is the area of the room surfaces.

TABLE 11.1 Impedances and modal densities for the baffled plate example

Octave band centre frequency (Hz)	$Z_{M\text{beam}}$	$\text{Re}\{Z_{M\text{plate}}\}$	$\text{Imag}\{Z_{M\text{plate}}\}$	n_{beam}	n_{plate}	n_{room}
63	12.74(1−j)	302	−1330	2.94E-3	2.30E-2	4.50E-2
125	9.04(1−j)	186	−734	2.09E-3	2.30E-2	0.137
250	6.39(1−j)	116	−407	1.48E-3	2.30E-2	0.467
500	4.52(1−j)	73.9	−227	1.04E-3	2.30E-2	1.71
1000	3.20(1−j)	48.7	−128	7.38E-4	2.30E-2	6.51
2000	2.26(1−j)	33.3	−72.9	5.21E-4	2.30E-2	25.4

Step 5: Coupling Loss Factors
The coupling loss factor for the beam to the plate (with number of connections, $q = 1$) can be calculated using Equations (11.87) and (11.88) in the textbook as:

$$\eta_{12}(\omega) = \frac{2}{\pi\omega n_{\text{beam}}(\omega)} \frac{\text{Re}(Z_{\text{beam}})\text{Re}(Z_{\text{plate}})}{|Z_{\text{beam}} + Z_{\text{plate}}|^2} \tag{11.15}$$

where the subscript 1 refers to the beam and the subscript 2 refers to the plate.

The coupling loss factor for the baffled panel to the room is given by Equation (11.97) in the textbook:

$$\eta_{23}(\omega) = \frac{\rho c S_p}{\omega m}\sigma = \frac{\rho c S_p}{\omega \rho h}\sigma = \frac{1.206 \times 343 \times 3 \times 3}{7850 \times 0.02}(\sigma/\omega) = 23.71(\sigma/\omega) \tag{11.16}$$

where the radiation efficiency, σ, of the panel is determined by the frequency and the dimensions of the panel. The fundamental resonance frequency of the panel is given by Equation (7.45) in the textbook and the critical frequency of the panel can be determined using Equations (7.3) and (7.5) in the textbook and the relation, $m = \rho_m h$. Thus:

$$f_{1,1} = 0.453 c_L h \left[\frac{1}{\ell_1^2} + \frac{1}{\ell_2^2}\right] = 0.453 \times 5383 \times 0.02(1/9 + 1/9) = 10.8 \text{ Hz} \tag{11.17}$$

$$f_c = \left(\frac{c^2}{2\pi}\right)\sqrt{\frac{12\rho(1-\nu^2)}{Eh^2}} = \left(\frac{343^2}{2\pi}\right)\sqrt{\frac{12 \times 7850 \times (1 - 0.3^2)}{207 \times 10^9 \times (20 \times 10^{-3})^2}} = 602 \text{ Hz} \tag{11.18}$$

The radiation efficiency results, calculated using Using Equations (4.176) to (4.182) in the textbook, and the coupling loss factors, calculated using Equations (11.87), (11.88) and (11.97), are listed in Table 11.2 below.

Step 6: Damping Loss Factors
The damping loss factors for the beam and plate are $\eta_1 = \eta_2 = 0.15$. The damping loss factor for the room, η_3, is given by Equation (11.77e) in the textbook, where it is assumed that the energy from the vibrating plate is transferred into the reverberant acoustic field in the room. Values of η_3 are frequency dependent and are included in Table 11.2 below.

TABLE 11.2 Radiation efficiencies and loss factors for the baffled plate example

Octave band centre frequency (Hz)	σ	η_{12}	η_{23}	η_3
63	0.0360	1.11×10^{-3}	2.15×10^{-3}	1.82×10^{-2}
125	0.0301	1.11×10^{-3}	9.09×10^{-4}	9.17×10^{-3}
250	0.0450	1.10×10^{-3}	6.79×10^{-4}	4.59×10^{-3}
500	0.266	1.08×10^{-3}	2.01×10^{-3}	2.29×10^{-3}
1000	1.6	1.07×10^{-3}	5.99×10^{-4}	1.15×10^{-3}
2000	1.2	1.06×10^{-3}	2.26×10^{-4}	5.73×10^{-4}

Step 7: Input Powers
The input power to the system is 1 W applied to the beam.

Step 8: System Equation and Solution
A matrix equation can be created as illustrated in Equations (11.57) to (11.59) in the textbook, and the energies in the subsystems calculated accordingly. The matrix equation is:

$$\omega \begin{bmatrix} (\eta_1+\eta_{12}+\eta_{13})n_1 & -\eta_{12}n_1 & -\eta_{13}n_1 \\ -\eta_{21}n_2 & (\eta_2+\eta_{21}+\eta_{23})n_2 & -\eta_{23}n_2 \\ -\eta_{31}n_3 & -\eta_{32}n_3 & (\eta_3+\eta_{31}+\eta_{32})n_3 \end{bmatrix} \begin{bmatrix} E_1/n_1 \\ E_2/n_2 \\ E_3/n_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{11.19}$$

Thus:

$$\begin{bmatrix} E_1/n_1 \\ E_2/n_2 \\ E_3/n_3 \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} (\eta_1+\eta_{12}+\eta_{13})n_1 & -\eta_{12}n_1 & -\eta_{13}n_1 \\ -\eta_{21}n_2 & (\eta_2+\eta_{21}+\eta_{23})n_2 & -\eta_{23}n_2 \\ -\eta_{31}n_3 & -\eta_{32}n_3 & (\eta_3+\eta_{31}+\eta_{32})n_3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{11.20}$$

This equation can be rewritten in a more convenient form using Equation (11.84) in the textbook and realising that $\eta_{13} = \eta_{31} = 0$. Thus:

$$\begin{bmatrix} E_1/n_1 \\ E_2/n_2 \\ E_3/n_3 \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} (\eta_1+\eta_{12})n_1 & -\eta_{12}n_1 & 0 \\ -\eta_{21}n_2 & (\eta_2+\eta_{21}+\eta_{23})n_2 & -\eta_{23}n_2 \\ 0 & -\eta_{32}n_3 & (\eta_3+\eta_{32})n_3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{11.21}$$

Substituting in appropriate values for the variables gives for the 63 Hz band:

$$\begin{bmatrix} E_1/0.00294 \\ E_2/0.0230 \\ E_3/0.0450 \end{bmatrix} = \frac{1}{395.8} \begin{bmatrix} (0.15+0.00111+0.00215)0.00294 & -0.00111 \times 0.00294 & 0 \\ -0.000142 \times 0.023 & (0.15+0.000142+0.00215)0.0230 & -0.00215 \times 0.0230 \\ 0 & -0.0011 \times 0.045 & (0.0182+0.0011)0.0450 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{11.22}$$

Thus:

$$\begin{bmatrix} E_1/0.00294 \\ E_2/0.0230 \\ E_3/0.0450 \end{bmatrix} = \frac{1}{395.8} \begin{bmatrix} 2252.2 & 2.0992 & 0.1201 \\ 2.0992 & 285.2 & 16.30 \\ 0.1201 & 16.315 & 1153.3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{11.23}$$

Thus, results for the 63 Hz band are::

$$L_{v,\mathrm{beam}} = 10\log_{10}\left(\frac{0.00294 \times 2252.2}{395.8 \times 7850 \times 3 \times 3 \times 0.02}\right) + 180 = 130.7 \text{ (dB re } 10^{-9}\text{m)} \tag{11.24}$$

$$L_{v,\mathrm{plate}} = 10\log_{10}\left(\frac{0.0230 \times 2.0992}{395.8 \times 7850 \times 3 \times 3 \times 0.02}\right) + 180 = 109.4 \text{ (dB re } 10^{-9}\text{m)} \tag{11.25}$$

$$L_p = 10\log_{10}\left(\frac{0.0450 \times 0.1201 \times 1.206 \times 343^2}{395.8 \times 125}\right) + 94 = 75.9 \text{ (dB re } 20\,\mu\text{Pa)} \tag{11.26}$$

Step 9: Amplitude Responses
Once the energy levels, E_i, in each subsystem, i, are determined, the vibration velocity of the beam and plate are calculated using Equation (11.60) in the textbook and the sound pressure level in the room is calculated using Equation (11.62) in the textbook giving the results in Table 11.3 below.

TABLE 11.3 Calculated velocities and sound pressure levels for Example 11.3

Octave band centre frequency (Hz)	SPL (dB re 20 µPa)	Velocity of beam (dB re 10^{-9} m/s)	Velocity of plate (dB re 10^{-9} m/s)
63	75.9	130.7	109.4
125	72.3	127.8	106.4
250	71.1	124.7	103.4
500	75.7	121.7	100.3
1000	80.3	118.7	97.1
2000	76.1	115.7	94.1

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